Name: ___________ **Course #: ______**

A2R

UNIT 3: LINEAR FUNCTIONS, EQUATIONS, AND THEIR ALGEBRA

FLIP VIDEO NOTES

- ☐ 3.1: Direct Variation
- ☐ 3.2: Average Rate of Change
- ☐ 3.3: Forms of a Line
- ☐ 3.4: Linear Modeling
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- ☐ 3.7: Systems of Linear Equations

3.1: DIRECT VARIATION

PROPORTIONAL OR DIRECT RELATIONSHIPS

Two variables, x and y, have a **direct (proportional) relationship** if for every ordered pair (x, y) we have:

$$\frac{y}{x} = k \text{ or } y = kx$$

Stated succinctly, y will always be a constant multiple of x. The value of k is known as the **constant of variation**.

***Exercise* #1:**
In each of the following, x and y are directly related.
Solve for the missing value.

$y = 15$ when $x = 5$

$y = ?$ when $x = 9$

You Try It! $y = 12$ when $x = 16$

$y = ?$ when $x = 24$

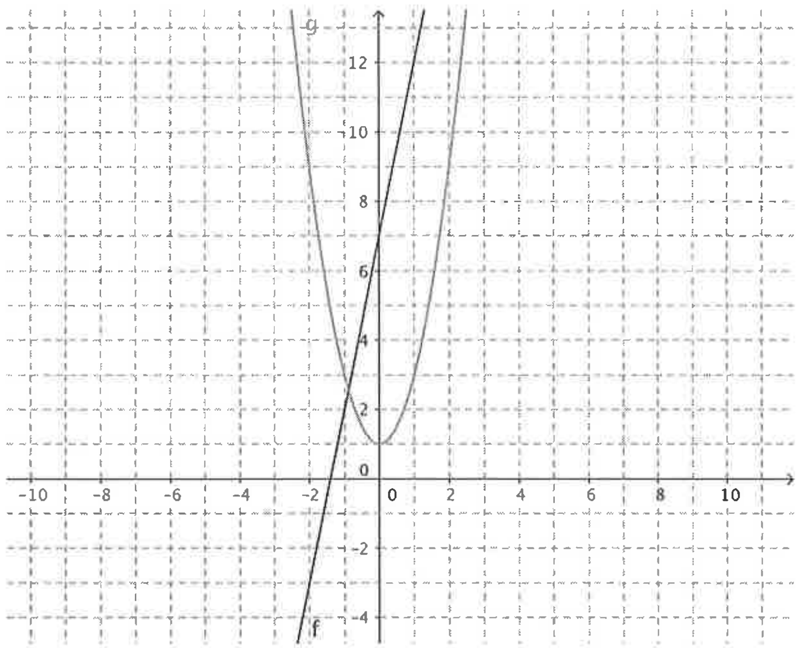
***Exercise* #2:** The distance a person can travel varies directly with the time they have been traveling if going at a constant speed. If Phoenix traveled 78 miles in 1.5 hours while going at a constant speed, how far will he travel in 2 hours at the same speed?

***You Try It!* :** Jenna works a job where her pay varies directly with the number of hours she has worked. In one week, she worked 35 hours and made \$274.75. How many hours would she need to work in order to earn \$337.55?

AVERAGE RATE OF CHANGE

For a function over the domain interval $a \le x \le b$, the function's **average rate of change** is calculated by:

$$\frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b)-f(a)}{b-a}$$



***Exercise* #1:** Consider the two functions $f(x)=5x+7$ and $g(x)=2x^2+1$.

Calculate the average rate of change for both functions over the following intervals. Do your work carefully and show the calculations that lead to your answers.

A. $-2 \le x \le 3$

B. You Try It! $1 \le x \le 5$

3.3: FORMS OF A LINE

TWO COMMON FORMS OF A LINE

Slope-Intercept: $y = mx + b$ **Point-Slope:** $y - y_1 = m(x - x_1)$

where m is the slope (or average rate of change) of the line and (x_1, y_1) represents one point on the line.

***Exercise* #1:** Consider a line whose slope is 5 and which passes through the point $(-2, 8)$.

(a) Write the equation of this line in point-slope form, $y - y_1 = m(x - x_1)$.

(b) Write the equation of this line in slope-intercept form, $y = mx + b$.

***You Try It!!*:** Write an equation for the line that is parallel to $y = \frac{3}{2}x - 7$ and which passes through the point $(6, -8)$?

***Exercise* #2:** A line passes through the points $(5, -2)$ and $(20, 4)$.

Write the equation for the

(a) Write the equation of this line in point-slope form, $y - y_1 = m(x - x_1)$.

(b) Write the equation of this line in slope-intercept form, $y = mx + b$.

You Try It!: A line passes through the points (12, 9) and (-2, 2).

(a) Write the equation of this line in point-slope form, $y - y_1 = m(x - x_1)$.

(b) Write the equation of this line in slope-intercept form, $y = mx + b$.

3.4: LINEAR MODELING

***Exercise* #1:** Dia was driving away from New York City at a constant speed of 58 miles per hour. He started 45 miles away.

(a) Write a linear function that gives Dia's distance, D, from New York City as a function of the number of hours, h, he has been driving.

(b) If Dia's destination is 270 miles away from New York City, algebraically determine to the nearest tenth of an hour how long it will take Dia to reach his destination.

***Exercise* #2:** Edelyn is trying to model her cell-phone plan. She knows that it has a fixed cost, per month, along with a \$0.15 charge per call she makes. In her last month's bill, she was charged \$12.80 for making 52 calls.

(a) Create a linear model, in point-slope form, for the amount Edelyn must pay, P, per month given the number of phone calls she makes, c.

(b) How much is Edelyn's fixed cost? In other words, how much would she have to pay for making zero phone calls?

3.5: INVERSES OF LINEAR FUNCTIONS

***Exercise* #1:** Find the the inverse of $y = 5x - 20$?

***You Try It!!* :**Find the Inverse of $y = \frac{2}{3}x + 8$?

***Exercise* #2:** Find an equation for the inverse of $y + 6 = 4(x - 2)$?

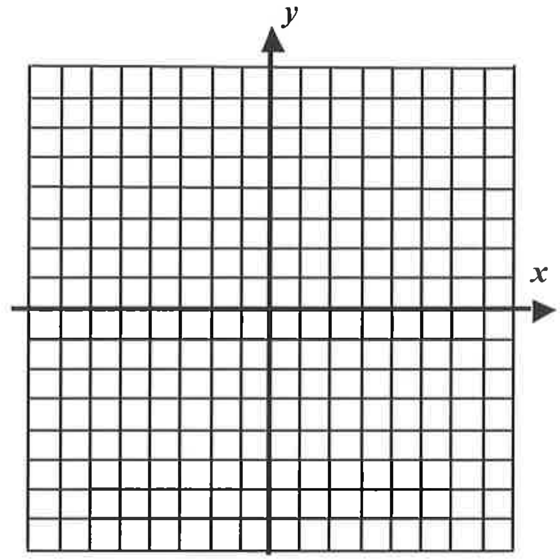
***You Try It!*:** Find the equation of the inverse of $y - 8 = 5(x + 2)$? Explain your choice.

3.6: PIECEWISE LINEAR FUNCTIONS

***Exercise* #1:** Consider the piecewise linear function given by the formula $f(x)=\begin{cases} x-3 & -3\le x<0 \\ \frac{1}{2}x+4 & 0\le x\le 4 \end{cases}$.

(a) Create a table of values below and graph the function.

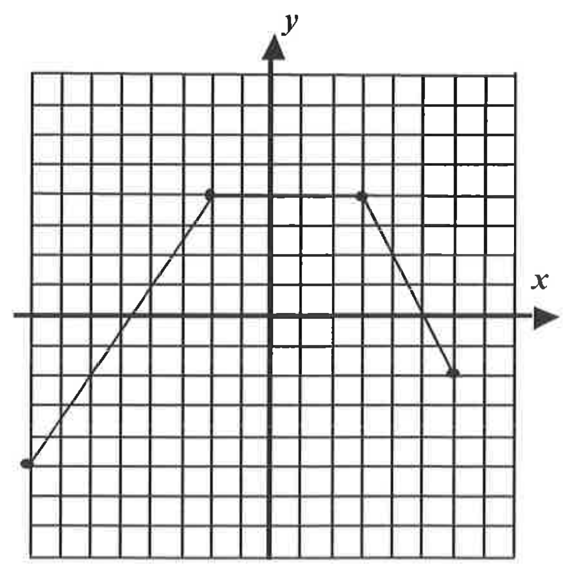
x	-3	-2	-1	0	1	2	3	4
$f(x)$								



(b) State the range of f using interval notation.

Not only should we be able to graph piecewise functions when we are given their equations, but we should also be able to translate the graphs of these functions into equations.

***Exercise* #2:** The function $f(x)$ is shown graphed below. Write a piecewise linear formula for the function. Be sure to specify both the formulas and the domain intervals over which they apply.



3.7: Systems of Linear Equations

***Exercise* #1:** Solve the below system of linear equations

$$2x + y + z = 15 \quad (1)$$
$$6x - 3y - z = 35 \quad (2)$$
$$-4x + 4y - z = -14 \quad (3)$$

(a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (1) and (3). Why is this an effective strategy in this case?

(b) Use this new two-by-two system to solve the three-by-three.

***Exercise* #2:** Consider the system of equations shown below. Answer the following questions based on the system.

$$4x + y - 3z = -6$$
$$-2x + 4y + 2z = 38$$
$$5x - y - 7z = -19$$

(a) Which variable will be easiest to eliminate? Why? Use the multiplicative property of equality and elimination to reduce this system to a two-by-two system.

(b) Solve the two-by-two system from (a) and find the final solution to the three-by-three system.

***You Try It!!*:** Solve the system of equations shown below. Show each step in your solution process.

$$\begin{aligned} 4x - 2y + 3z &= 23 \\ x + 5y - 3z &= -37 \\ -2x + y + 4z &= 27 \end{aligned}$$