Name: _____ Course #: _____

A2R

UNIT 3: LINEAR FUNCTIONS, EQUATIONS, AND THEIR ALGEBRA

CLASS WORK PROBLEM SETS

- ☐ 3.1: Direct Variation
- ☐ 3.2: Average Rate of Change
- ☐ 3.3: Forms of a Line
- ☐ 3.4: Linear Modeling
- ☐ 3.5: Inverses of Linear Functions
- ☐ 3.6: Piecewise Linear Functions
- ☐ 3.7: Systems of Linear Equations

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Algebra 2R Unit 3: Linear Functions, Equations, and Their Algebra

3.1 Problem Set – Direct Variation

FLUENCY

1. In each of the following, the variable pair given **are proportional** to one another. Find the missing value.

   (a) $b = 8$ when $a = 16$
   $b = ?$ when $a = 18$

   (b) $y = 10$ when $x = 14$
   $y = ?$ when $x = 21$

   (c) $w = -2$ when $u = 6$
   $w = ?$ when $u = -15$

2. In the following exercises, the two variables given **vary directly** with one another. Solve for the missing value.

   (a) $p = 12$ when $q = 8$
   $p = ?$ when $q = 6$

   (b) $y = 21$ when $x = 9$
   $y = ?$ when $x = -6$

   (c) $z = -5$ when $w = 2$
   $z = ?$ when $w = 8$

3. If x and y vary directly and $y = 16$ when $x = 12$, then which of the following equations correctly represents the relationship between x and y?

   (1) $y = \frac{3}{4}x$

   (2) $y + x = 28$

   (3) $xy = 192$

   (4) $y = \frac{4}{3}x$

APPLICATIONS

4. The distance Max's bike moves is directly proportional to how many rotations his bike's crank shaft has made. If Max's bike moves 25 feet after two rotations, how many feet will the bike move after 15 rotations?

5. For his workout, the increase in Jacob's heart rate is directly proportional to the amount of time he has spent working out. If his heartbeat has increased by 8 beats per minute after 20 minutes of working out, how much will his heartbeat have increased after 30 minutes of working out?

6. When a photograph is enlarged or shrunken, its width and length stay proportional to the original width and length. Rojas is enlarging a picture whose original width was 3 inches and whose original length was 5 inches. If its new length is to be 8 inches, what is the exact value of its new width in inches?

7. For a set amount of time, the distance Kirk can run is directly related to his average speed. If Kirk can run 3 miles in while running at 6 miles per hour, how far can he run in the same amount of time if his speed increases to 10 miles per hour?

REASONING

8. Two variables are proportional if they can be written at $y = kx$, where k is some constant. This leads to the fact that when $x = 0$ then $y = 0$ as well. Is the temperature measured in Celsius proportional to the temperature measured in Fahrenheit? Explain.

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3.2 Problem Set – Average Rate of Change

FLUENCY

1. For the function $g(x)$ given in the table below, calculate the average rate of change for each of the following intervals.

x	-3	-1	4	6	9
$g(x)$	8	-2	13	12	5

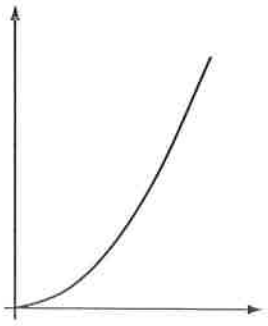
(a) $-3 \le x \le -1$ (b) $-1 \le x \le 6$ (c) $-3 \le x \le 9$

(d) Explain how you can tell from the answers in (a) through (c) that this is **not** a table that represents a linear function.

2. Consider the simple quadratic function $f(x) = x^2$. Calculate the average rate of change of this function over the following intervals:

(a) $0 \le x \le 2$ (b) $2 \le x \le 4$ (c) $4 \le x \le 6$

(d) Clearly the average rate of change is getting larger at x gets larger. How is this reflected in the graph of f shown sketched to the right?



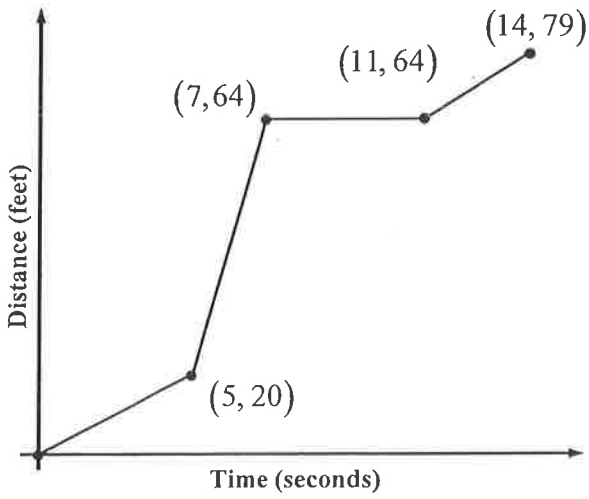
3. Which has a greater average rate of change over the interval $-2 \le x \le 4$, the function $g(x) = 16x - 3$ or the function $f(x) = 2x^2$? Provide justification for your answer.

APPLICATIONS

4. An object travels such that its distance, *d*, away from its starting point is shown as a function of time, *t*, in seconds, in the graph below.

   (a) What is the average rate of change of *d* over the interval $5 \le t \le 7$? Include proper units in your answer.

   (b) The average rate of change of distance over time (what you found in part (a)) is known as the **average speed** of an object. Is the average speed of this object greater on the interval $0 \le t \le 5$ or $11 \le t \le 14$? Justify.



REASONING

5. What makes the average rate of change of a linear function different from that of any other function? What is the special name that we give to the average rate of change of a linear function?

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Algebra 2R Unit 3: Linear Functions, Equations, and Their Algebra

3.3 Problem Set – Forms of a Line

FLUENCY

1. Which of the following lines is *perpendicular* to $y=\frac{5}{3}x-7$ and has a y-intercept of 4?

   (1) $y=\frac{5}{3}x+4$ (3) $y=4x-\frac{3}{5}$

   (2) $y=-\frac{3}{5}x+4$ (4) $y=\frac{3}{5}x+4$ _____

2. Which of the following lines passes through the point $(-4,-8)$?

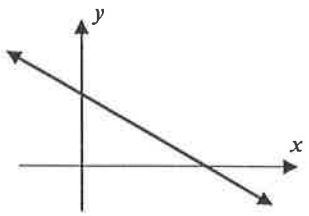
   (1) $y+8=3(x+4)$ (3) $y+8=3(x-4)$

   (2) $y-8=3(x-4)$ (4) $y-8=3(x+4)$ _____

3. Which of the following equations could describe the graph of the linear function shown below?

   (1) $y=\frac{2}{3}x-4$ (3) $y=-\frac{2}{3}x-4$

   (2) $y=\frac{2}{3}x+4$ (4) $y=-\frac{2}{3}x+4$



_____

4. For a line whose slope is -3 and which passes through the point $(5,-2)$:

   (a) Write the equation of this line in point-slope form, $y-y_1=m(x-x_1)$.

   (b) Write the equation of this line in slope-intercept form, $y=mx+b$.

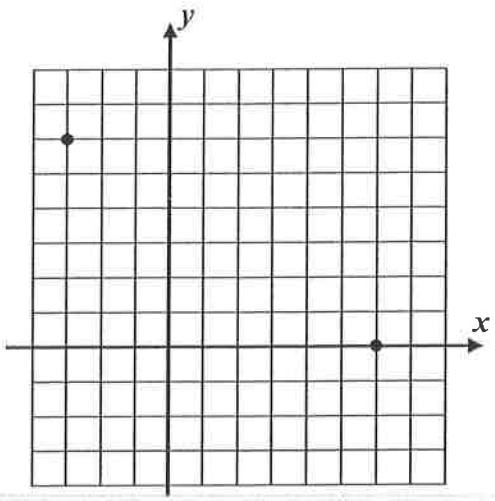
5. For a line whose slope is 0.8 and which passes through the point $(-3,1)$:

   (a) Write the equation of this line in point-slope form, $y-y_1=m(x-x_1)$.

   (b) Write the equation of this line in slope-intercept form, $y=mx+b$.

REASONING

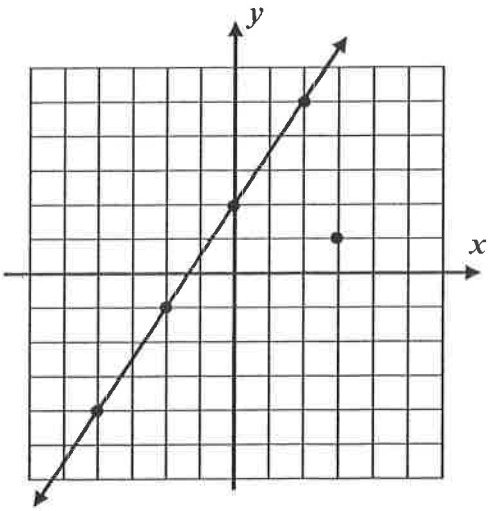
6. The two points $(-3, 6)$ and $(6, 0)$ are plotted on the grid below.



(a) Find an equation, in $y = mx + b$ form, for the line passing through these two points. Use of the grid is optional.

(b) Does the point $(30, -16)$ lie on this line? Justify.

7. A linear function is graphed below along with the point $(3, 1)$.



(a) Draw a line parallel to the one shown that passes through the point $(3, 1)$.

(b) Write an equation for the line you just drew in point-slope form.

(c) Between what two consecutive integers does the y-intercept of the line you drew fall?

(d) Determine the *exact* value of the y-intercept of the line you drew.

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3.4 Problem Set – Linear Modeling

APPLICATIONS

1. Which of the following would model the distance, D, a driver is from Chicago if they are heading *towards* the city at 58 miles per hour and started 256 miles away?

   (1) $D = 256t + 58$ (3) $D = 58t + 256$

   (2) $D = 256 - 58t$ (4) $D = 58 - 256t$

   _____

2. The cost, C, of producing x-bikes is given by $C = 22x + 132$. The revenue gained from selling x-bikes is given by $R = 350x$. If the profit, P, is defined as $P = R - C$, then which of the following is an equation for P in terms of x?

   (1) $P = 328x - 132$ (3) $P = 328x + 132$

   (2) $P = 372x + 132$ (4) $P = 372x - 132$

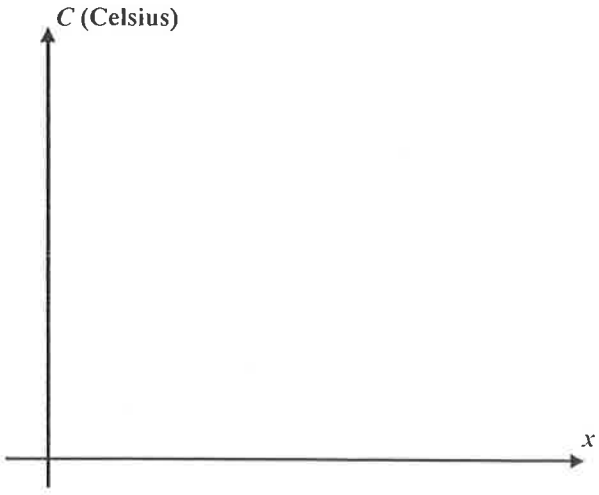
   _____

3. The average temperature of the planet is expected to rise at an average rate of 0.04 degrees Celsius per year due to global warming. The average temperature in the year 2000 was 14.71 degrees Celsius. The average Celsius temperature, C, is given by $C = 14.71 + 0.04x$, where x represents the number of years since 2000.

   (a) What will be the average temperature in the year 2100?

   (b) Algebraically determine the number of years, x, it will take for the temperature, C, to reach 20 degrees Celsius. Round to the nearest year.

   (c) Sketch a graph of the average yearly temperature below for the interval $0 \le x \le 200$. Be sure to label your y-axis scale as well as two points on the line (the y-intercept and one additional point).

   (d) What does this model project to be the average global temperature in 2200?

4. Fabio is driving west away from Albany and towards Buffalo along Interstate 90 at a constant rate of speed of 62 miles per hour. After driving for 1.5 hours, Fabio is 221 miles from Albany.

(a) Write a linear model for the distance, D, that Fabio is away from Albany as a function of the number of hours, h, that he has been driving. Write your model in point-slope form, $D - D_1 = m(h - h_1)$.

(b) Rewrite this model in slope-intercept form, $D = mh + b$.

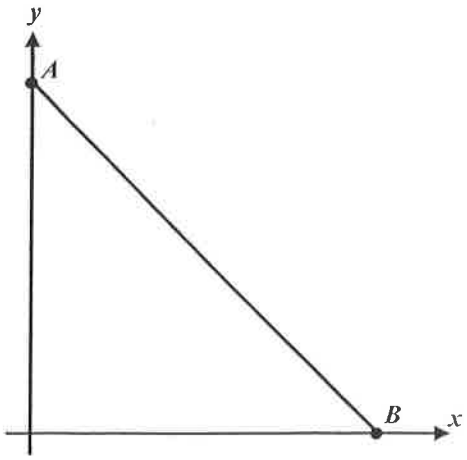
(c) How far was Fabio from Albany when he started his trip?

(d) If the total distance from Albany to Buffalo is 290 miles, determine how long it takes for Fabio to reach Buffalo. Round your answer to the nearest tenth of an hour.

5. A particular rocket taking off from the Earth's surface uses fuel at a constant rate of 12.5 gallons per minute. The rocket initially contains 225 gallons of fuel.

(a) Determine a linear model, in $y = ax + b$ form, for the amount of fuel, y, as a function of the number of minutes, x, that the rocket has burned.

(b) Below is a general sketch of what the graph of your model should look like. Using your calculator, determine the x and y intercepts of this model and label them on the graph at points A and B respectively.



(c) The rocket must still contain 50 gallons of fuel when it hits the stratosphere. What is the maximum number of minutes the rocket can take to hit the stratosphere? Show this point on your graph by also graphing the horizontal line $y = 50$ and showing the intersection point.

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3.5 Problem Set – Inverses of Linear Functions

FLUENCY

1. The graph of a function and its inverse are always symmetric across which of the following lines?

   (1) $y=0$ (3) $y=x$

   (2) $x=0$ (4) $y=1$

2. Which of the following represents the inverse of the linear function $y=3x-24$?

   (1) $y=\frac{1}{3}x+8$ (3) $y=-\frac{1}{3}x+24$

   (2) $y=-\frac{1}{3}x-8$ (4) $y=\frac{1}{3}x-\frac{1}{24}$

3. If the y-intercept of a linear function is 8, then we know which of the following about its inverse?

   (1) Its y-intercept is -8. (3) Its y-intercept is $\frac{1}{8}$.

   (2) Its x-intercept is 8. (4) Its x-intercept is -8.

4. If both were plotted, which of the following linear functions would be parallel to its inverse? Explain your thinking.

   (1) $y=2x$ (3) $y=5x-1$

   (2) $y=\frac{2}{3}x-4$ (4) $y=x+6$

5. Which of the following represents the equation of the inverse of $y=\frac{4}{3}x+24$?

   (1) $y=-\frac{4}{3}x-24$ (3) $y=\frac{3}{4}x-18$

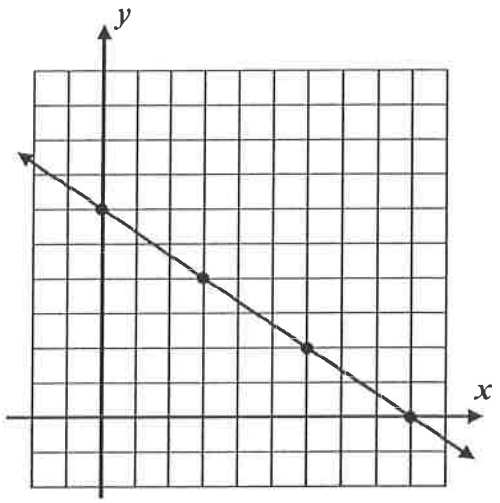
   (2) $y=-\frac{3}{4}x+18$ (4) $y=\frac{4}{3}x-24$

6. Which of the following points lies on the inverse of $y+2=4(x-1)$?

   (1) $(2,-1)$ (3) $\left(\frac{1}{2},1\right)$

   (2) $(-1,2)$ (4) $(-2,1)$

7. A linear function is graphed below. Answer the following questions based on this graph.



(a) Write the equation of this linear function in $y = mx + b$ form.

(b) Sketch a graph of the inverse of this function on the same grid.

(c) Write the equation of the inverse in $y = mx + b$ form.

(d) What is the intersection point of this line with its inverse?

APPLICATIONS

8. A car traveling at a constant speed of 58 miles per hour has a distance of y-miles from Poughkeepsie, NY, given by the equation $y = 58x + 24$, where x represents the time in hours that the car has been traveling.

(a) Find the equation of the inverse of this linear function in $y = \frac{x - a}{b}$ form.

(b) Evaluate the function you found in part (a) for an input of $x = 227$.

(c) Give a physical interpretation of the answer you found in part (b). Consider what the input and output of the inverse represent in order to answer this question.

REASONING

9. Given the general linear function $y = mx + b$, find an equation for its inverse in terms of m and b.

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Algebra 2R Unit 3: Linear Functions, Equations, and Their Algebra

3.6 Problem Set – Piecewise Linear Functions

FLUENCY

1. For $f(x)=\begin{cases} 5x-3 & x<-2 \\ x+8 & -2\le x<3 \\ \frac{1}{3}x+7 & x\ge 3 \end{cases}$ answer the following questions.

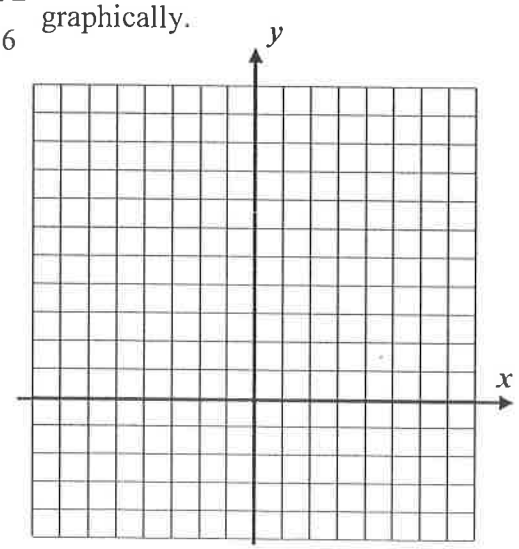
   (a) Evaluate each of the following by carefully applying the correct formula:

   (i) $f(2)$ (ii) $f(-4)$ (iii) $f(3)$ (iv) $f(0)$

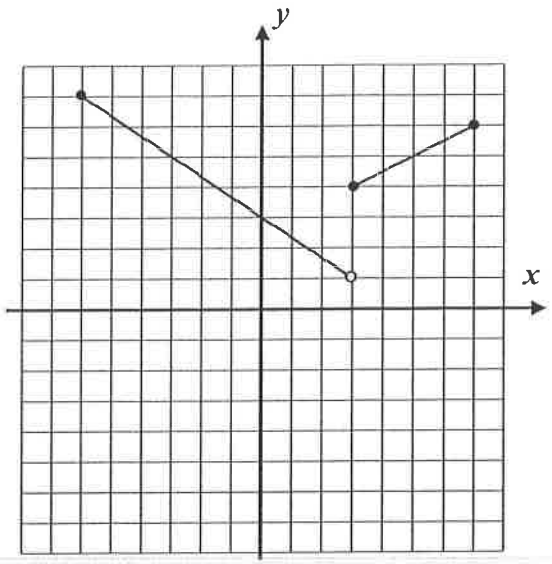
   (b) The three linear equations have y-intercepts of $-3, 8$ and 7 respectively. Yet, a function can have only one y-intercept. Which of these is the y-intercept of this function? Explain how you made your choice.

   (c) Calculate the average rate of change of f over the interval $-3\le x\le 9$. Show the calculations that lead to your answer.

2. Determine the range of the function $g(x)=\begin{cases} x+4 & -2\le x\le 2 \\ -\frac{3}{2}x+9 & 2<x\le 6 \end{cases}$ graphically.



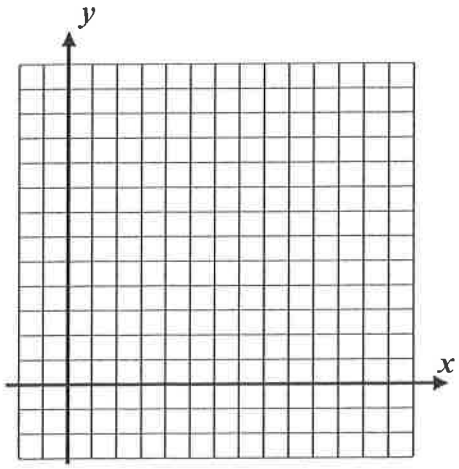
3. Determine a piecewise linear equation for the function $f(x)$ shown below. Be sure to specify not only the equations, but also the domain intervals over which they apply.



REASONING

4. Step functions are piecewise functions that are constants (horizontal lines) over each part of their domains. Graph the following step function.

$$f(x)=\begin{cases}-2 & 0\le x<3\\ 3 & 3\le x<5\\ 7 & 5\le x<10\\ 5 & 10\le x\le 12\end{cases}$$



5. Find all x-intercepts of the function $g(x)=\begin{cases}2x+8 & -5\le x<-1\\ -\frac{1}{2}x-4 & -1\le x<1\\ -4x+10 & 1\le x\le 4\end{cases}$ algebraically. Justify your work by showing your algebra. Be sure to check your answers versus the domain intervals to make sure each solution is valid.

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Algebra 2R Unit 3: Linear Functions, Equations, and Their Algebra

3.7 Problem Set – Piecewise Linear Functions

FLUENCY

1. The sum of two numbers is 5 and the larger difference of the two numbers is 39. Find the two numbers by setting up a system of two equations with two unknowns and solving algebraically.

2. Algebraically, find the intersection points of the two lines whose equations are shown below.

$$4x+3y=-13$$
$$y=6x-8$$

3. Show that $x=10$, $y=4$, and $z=7$ is a solution to the system below *without* solving the system formally.

$$x+2y+z=25$$
$$4x-y-5z=1$$
$$-2x-y+8z=32$$

4. In the following system, the value of the constant c is unknown, but it is known that $x=-8$ and $y=4$ are the x and y values that solve this system. Determine the value of c. Show how you arrived at your answer.

$$-5x+2y+3z=81$$
$$x-y+z=-1$$
$$2x-y+cz=35$$

5. Solve the following system of equations. Carefully show how you arrived at your answers.

$$\begin{aligned} 4x+2y-z &= 21 \\ -x-2y+2z &= 13 \\ 3x-2y+5z &= 70 \end{aligned}$$

6. Algebraically solve the following system of equations. There are two variables that can be readily eliminated, but your answers will be the same no matter which you eliminate first.

$$\begin{aligned} 2x+5y-z &= -35 \\ x-3y+4z &= 31 \\ -3x+2y+2z &= -23 \end{aligned}$$

7. Algebraically solve the following system of equations. This system will take more manipulation because there are no variables with coefficients equal to 1.

$$\begin{aligned} 2x+3y-2z &= 33 \\ 4x+5y+3z &= 54 \\ -6x-2y-8z &= -50 \end{aligned}$$