A2R

UNIT 10: POLYNOMIAL AND RATIONAL FUNCTIONS

PROBLEM SETS

Information in parenthesis is ixl.com reference for practice or location of practice worksheets on the topic.

10.1: Power Functions
10.2: Graphs and Zeros of a Polynomial (A2: K.1, K.8, PC: D.11)
10.3: Creating Polynomial Equations (A2: K.8, K.9, PC: D.5)
10.4: Polynomial Identities (Google Drive "Polynomial Identities" Folder)
10.5: Introduction to Rational Functions (A2: N.1)
10.6: Simplifying Rational Expressions (A2: N.4)
10.7: Multiplying and Dividing Rational Expressions (A2: N.5)
10.8: Combining Rational Expressions Using Addition and Subtraction (A2: N.6)
10.9: Complex Fractions (Google Drive "Complex Fractions" Folder)
10.10: Polynomial Long Division (A2: K.4, PC: D.1)
10.11: The Remainder Theorem (Class Google Drive "The Remainder Theorem" Folder)
10.12: Solving Rational Equations (A2: N.7)
10.13: Solving Rational Inequalities (Google Drive "Solving Rational Inequalities" Folder)
10.14: Reasoning About Radical and Rational Equations

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Algebra 2R Unit 9: Complex Numbers

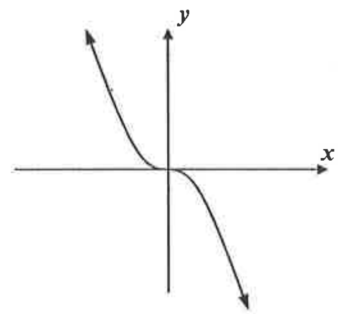
10.1 Problem Set – Power Functions

FLUENCY

1. **Without** using your calculator, determine which of the following equations could represent the graph shown below. Explain your choice.

(1) $y = x^2$ (2) $y = x^3$

(3) $y = -x^4$ (4) $y = -x^5$



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2. Identify which of the following are power functions. For each that is a power function, write it in the form $y = ax^n$, where a and n are real numbers. Placing them in these forms may take some mindful algebraic manipulation.

(a) $y = 3\sqrt[5]{x}$ (b) $y = 4x^5 - 7$ (c) $y = \dfrac{10}{x^5}$ (d) $y = \dfrac{6x^7}{2x^3}$

(e) $y = x^2 + 2x - 7$ (f) $y = \sqrt{48x^7}$ (g) $y = \sqrt{\dfrac{25}{x^4}}$ (h) $y = 2(x-3)^2$

3. If the point (-3, 8) lies on the graph of a power function with an even exponent, which of the following points must also lie its graph?

(1) $(3, -8)$ (3) $(-3, -8)$

(2) $(3, 8)$ (4) $(8, -3)$

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4. If the point $(-5,12)$ lies on the graph of a power function with an odd exponent, which of the following points must also lie on its graph?

(1) $(5,-12)$ (2) $(12,-5)$

(3) $(-5,-12)$ (4) $(-12,5)$

5. For each of the following polynomials, give a power function that best represents the end behavior of the polynomial.

(a) $y=3x^3-2x+12$ (b) $y=10-8x^2$ (c) $y=6x^5-4x^3+x-120$

(d) $y=-3x^5+2x^4-4x+7$ (e) $y=5x^4+2x^2$ (f) $y=-4x^5+8x^7-2x^3+3$

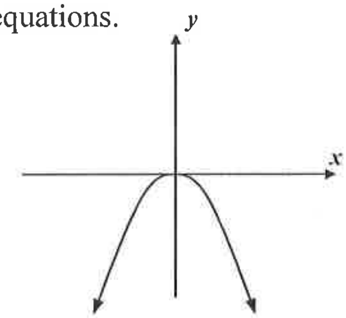
6 The graph below could be the long-run behavior for which of the following functions? Do this problem **without** graphing each of the following equations.

(1) $y=2x^2-7x+1$

(2) $y=4x^3+2x^2-6x+4$

(3) $y=-5x^4+3x^3-2x^2+x+9$

(4) $y=-3x^5-4x^2+2x+1$



REASONING

7. Let's examine why end-behavior works a little more closely. Consider the functions $f(x)=x^3$ and $g(x)=x^3+2x^2+7x+10$.

(a) Fill out the table below for the values of x listed. Round your final column to the nearest *hundredth*.

x	$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
5			
10			
50			
100			

(b) What number is the ratio in the fourth column approaching as x gets larger? What does this tell you about the part of $g(x)$ that can be attributed to the cubic term?

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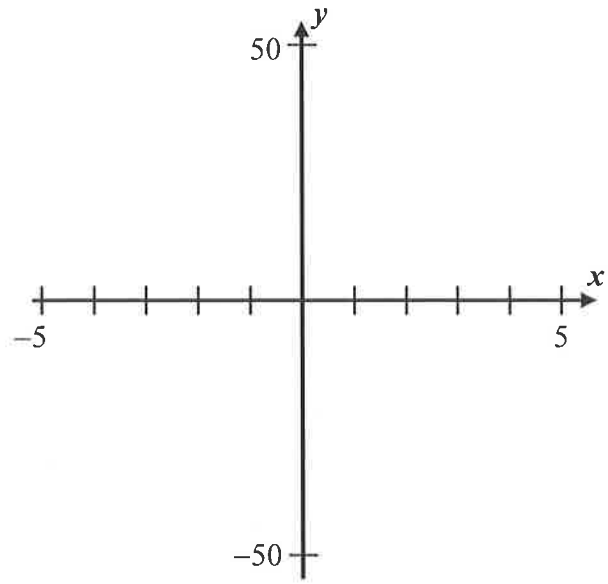
10.2 Problem Set – Graphs and Zeroes of a Polynomial

FLUENCY

1. Consider the cubic function $y=x^3+2x^2-8x$.

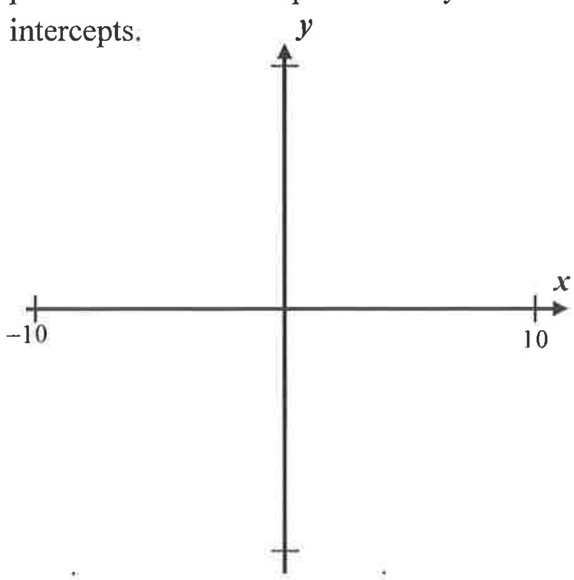
   (a) Algebraically determine the zeroes of this cubic function.

   (b) Sketch the function on the axes given. Clearly plot and label each x-intercept.



2. Consider the cubic function $y=x^3+2x^2-36x-72$.

   (a) Find an appropriate y-window for the x-window shown on the axes and sketch the graph. Make the sure the window is sufficiently large to show the two turning points and all intercepts. Clearly label all x-intercepts.

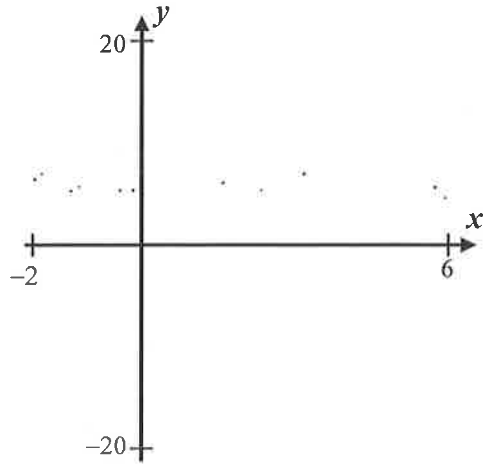


   (b) What are the solutions to the equation $x^3+2x^2-36x-72=0$?

   (c) Based on your answers to (b), how must the expression $x^3+2x^2-36x-72$ factor?

3. Consider the cubic function given by $y = x^3 - 6x^2 + 12x - 5$.

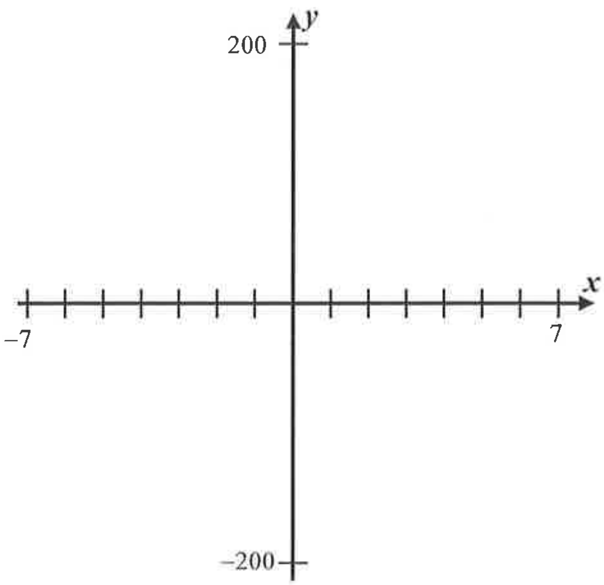
(a) Sketch a graph of this function on the axes given below.



(b) Considering the graphs of cubics you saw in class and those in problems 1 and 2, what is different about the way this cubic's graph looks compared to the others?

4. Consider the quartic function $y = x^4 - x^3 - 27x^2 + 25x + 50$.

(a) Sketch the graph of this function on the axes given below. Clearly mark all x-intercepts.



(b) Use your graph from part (a) to solve the equation $x^4 - x^3 - 27x^2 + 25x + 50 = 0$.

(c) Considering your answer to (b), how does the expression $x^4 - x^3 - 27x^2 + 25x + 50$ factor?

5. In general, how does the number of zeroes (or x-intercepts) relate to the highest power of a polynomial? Be specific. Can you make a statement about the minimum number of zeroes as well as the maximum?

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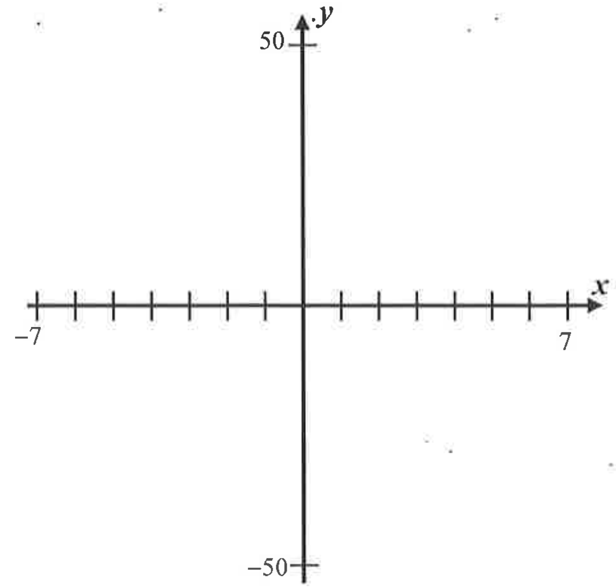
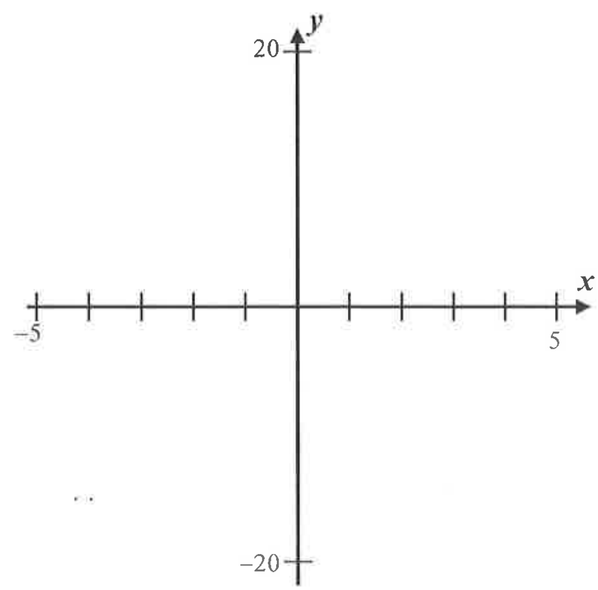
10.3 Problem Set – Creating Polynomial Equations

FLUENCY

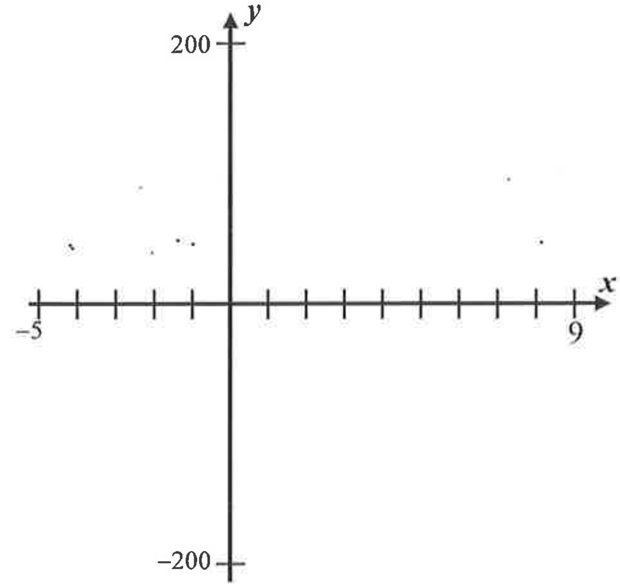
1. Create the equation of a quadratic polynomial, in standard form, that has zeroes of -5 and 2 and which passes through the point $(3,-24)$. Sketch the graph of the quadratic below to verify your result.



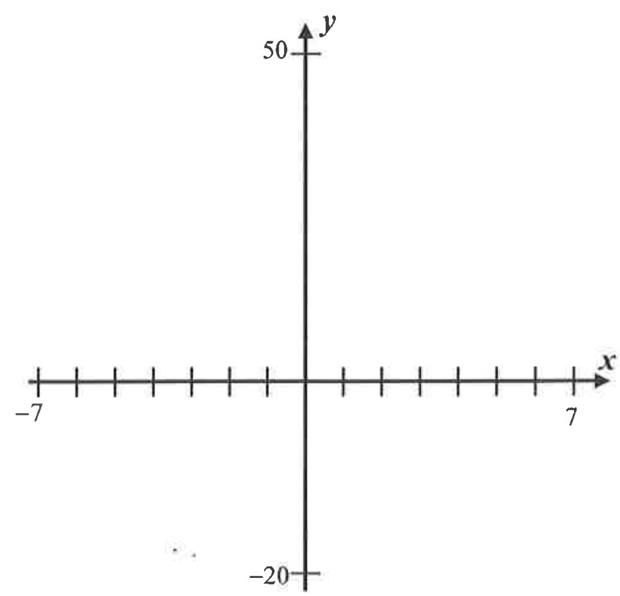
2. Create the equation of a quadratic function, in standard form, that has one zero of -3 and a turning point at $(-1,-16)$. Hint – try to determine the second zero of the parabola by thinking about the relationship between the first zero and the turning point (axis of symmetry). Sketch your solution below.



3. Create an equation for a cubic function, in standard form, that has x-intercepts given by the set $\{-3, 1, 7\}$ and which passes through the point $(-2, 54)$. Sketch your result on the axes shown below.



4. Create the equation of a cubic whose x-intercepts are given by the set $\{-6, -3, 5\}$ and which passes through the point $(3, 36)$. Note that your leading coefficient in this case will be a non-integer. Sketch your result below.



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10.4 Problem Set – Polynomial Identities

FLUENCY

1. One of the two expressions below is an identity and one of them is not. Determine which is an identity by testing the truth value of the equation for various values of x. Show the values of x that you test. Remember, an identity will be true for **every value of *x*.**

Equation #1: $(x+1)^2 = 4x+4$

Equation #2: $(x+2)^2 = x^2+4x+4$

2. Which of the following equations represents an identity?

(1) $2x+1=3x+4$

(2) $6x+3=2x+10$

(3) $4x-3=2(2x+7)$

(4) $4(5x+2)=20x+8$

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3. One of the more useful identities that students almost inherently learn is:

$$(x+c)(x+d)=x^2+(c+d)x+cd$$

(a) Prove this identity. You may choose to algebraically manipulate one or both sides of the equation to justify the equivalence.

(b) This identity allows you to multiply common binomials very quickly. Find the following products in simplest trinomial form.

(i) $(x+3)(x+7)$

(ii) $(x-7)(x-2)$

(iii) $(x+10)(x-3)$

4. You should be well aware of the difference of perfect squares, i.e. $x^2 - y^2 = (x-y)(x+y)$. But there is also an identity for the difference of perfect cubes:

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

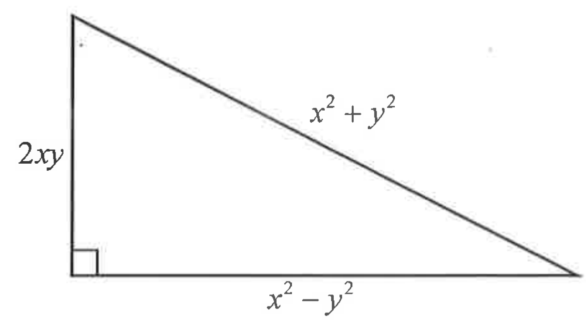
(a) Prove this identity by expanding the product on the right-hand side of the equation.

(b) Use the identity to find the value of $10^3 - 9^3$ without the use of your calculator. Show the steps in your calculation. Then, verify with your calculator.

APPLICATIONS

5. Another famous identity that can be used to generate Pythagorean Triples is shown below:

$$(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$$



The complicated sides are shown on the diagram.

(a) Prove this identity by expanding the products on both sides of the equation.

(b) Generate the sides of the right triangle if $x = 4$ and $y = 1$. Show that these sides satisfy the Pythagorean Theorem.

(c) **Reasoning:** This relationship leads to the conclusion that there is *no* Pythagorean triple of this form that contains the integer 2. Why?

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10.5 Problem Set – Introduction to Rational Functions

FLUENCY

1. Which of the following values of x is *not* in the domain of $f(x)=\dfrac{x+3}{x-7}$?

(1) $x=-7$ (3) $x=3$

(2) $x=7$ (4) $x=-3$

2. Which of the following values of x is *not* in the domain of $g(x)=\dfrac{4x-1}{2x+1}$?

(1) $x=-\dfrac{1}{2}$ (3) $x=\dfrac{1}{4}$

(2) $x=-1$ (4) $x=-3$

3. Which values of x, when substituted into the function $y=\dfrac{x-4}{2x^2+8x}$, would make it undefined?

(1) $x=2$ and 8 (3) $x=-4$ and 4

(2) $x=-4$ and 8 (4) $x=-4$ and 0

4. Which of the following represents the domain of $y=\dfrac{x^2-4}{x^2+5x-14}$?

(1) $\{x\,|\,x\neq\pm2\}$ (3) $\{x\,|\,x\neq-4 \text{ and } 14\}$

(2) $\{x\,|\,x\neq-7 \text{ and } 2\}$ (4) $\{x\,|\,x\neq-5 \text{ and } 14\}$

5. Which of the following represents the domain of $g(x)=\dfrac{3x-1}{2x^2-x-10}$?

(1) $\left\{x\,|\,x\neq\dfrac{1}{3}\right\}$ (3) $\left\{x\,|\,x\neq-\dfrac{1}{2} \text{ and } 5\right\}$

(2) $\left\{x\,|\,x\neq-\dfrac{1}{3} \text{ and } \dfrac{1}{2}\right\}$ (4) $\left\{x\,|\,x\neq-2 \text{ and } \dfrac{5}{2}\right\}$

6. If $f(x)=2x+7$ and $g(x)=\dfrac{x^2-4}{2x+1}$ then $g(f(-5))=?$

(1) -1 (3) 6

(2) 2 (4) -3

7. If $f(x)=\frac{3x-2}{2x}$ and $g(x)=4x-1$ then $f(g(x))=?$

(1) $\frac{7x-3}{2x}$ (3) $\frac{12x-5}{8x-2}$

(2) $\frac{12x-9}{8x-2}$ (4) $\frac{5x-4}{x}$

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8. The y-intercept of the rational function $y=\frac{2x+15}{x-3}$ is

(1) 15 (3) −3

(2) −5 (4) 12

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9. Find formulas for the inverse of each of the following rational functions.

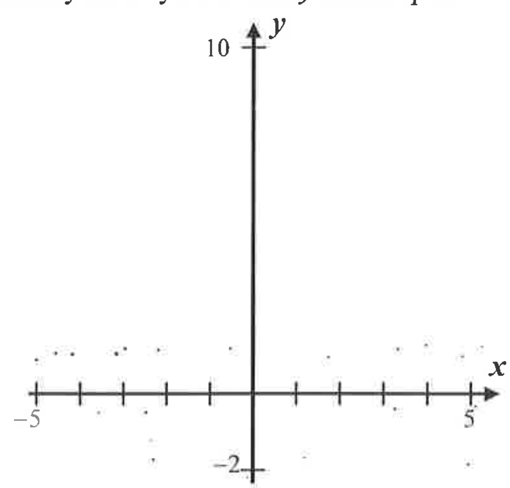
(a) $y=\frac{5x}{x-2}$ (b) $y=\frac{3x+2}{x+4}$

10. Consider the rational function $y=\frac{9-x^2}{x^2+1}$.

(a) Find the function's y-intercept algebraically.

(b) Find the function's x-intercepts algebraically.

(c) Sketch the function on the axes below. Clearly label your x and y intercepts.



(d) Is this an even or an odd function? Explain graphically.

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10.6 Problem Set – Simplifying Rational Expressions

FLUENCY

1. Write each of the following ratios in simplest form.

(a) $\frac{5x^8}{20x^2}$ (b) $\frac{-12y^3}{8y^{12}}$ (c) $\frac{6x^{10}y^2}{15x^4y^5}$ (d) $\frac{24x^3y^7}{12x^6y^{10}}$

2. Which of the following is equivalent to the expression $\frac{4x^6y^4}{12x^2y^6}$?

(1) $\frac{x^4}{3y^2}$ (3) $\frac{3x^3}{y^2}$

(2) $\frac{3y^2}{x^3}$ (4) $\frac{x^3}{3y^2}$

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3. Simplify each of the following rational expressions.

(a) $\frac{x^2-25}{4x-20}$ (b) $\frac{x^2+11x+24}{x^2-9}$ (c) $\frac{4x^2-1}{5x-10x^2}$

(d) $\frac{9x^2-4}{3x^2+4x-4}$ (e) $\frac{7x^2-42x}{x^2+2x-48}$ (f) $\frac{2x^2-3x-5}{25-4x^2}$

4. Which of the following is equivalent to the fraction $\frac{x^2-9x+18}{15x-5x^2}$?

(1) $\frac{x-3}{5x}$ (3) $\frac{6-x}{5x}$

(2) $\frac{x+6}{5x}$ (4) $\frac{-x-6}{5x}$

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5. The rational expression $\frac{2x^2+7x+6}{x^2-4}$ can be equivalently rewritten as

(1) $\frac{2x+3}{x-2}$ (3) $\frac{2x-3}{2-x}$

(2) $\frac{2x+1}{x-6}$ (4) $\frac{3-2x}{x+2}$

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6. Written in simplest form, the fraction $\frac{y^2-x^2}{5x-5y}$ is equal to

(1) $5y-5x$ (3) $\frac{-(x+y)}{5}$

(2) $\frac{y-x}{5}$ (4) $\frac{x-y}{5}$

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REASONING

7. When we simplify an algebraic fraction, we are producing equivalent expressions for *most* values of x. Consider the expressions $\frac{x^2-4}{2x-4}$ and $\frac{x+2}{2}$.

(a) Show by simplifying the first expression that these two are equivalent.

(b) Use your calculator to fill out the value for both of these expressions to show their equivalence.

x	$\frac{x^2-4}{2x-4}$	$\frac{x+2}{2}$
0		
1		
2		
3		
4		

(c) Clearly these two expressions are *not* equivalent for an input value of $x=2$. Explain why.

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10.7 Problem Set – Multiplying and Dividing Rational Expressions

SKILLS

1. Express each of the following products in simplest form.

(a) $\frac{12x^4}{5y^8} \cdot \frac{15y}{30x^2}$

(b) $\frac{14a^2}{15b^9} \cdot \frac{10b^3}{21a^6}$

(c) $\frac{4x^3}{9z^5} \cdot \frac{3y^7}{10x^2} \cdot \frac{30z^2}{8y^3}$

2. Write each of the following products in simplest form.

(a) $\frac{9x^2-16}{12x+16} \cdot \frac{8x+8}{3x^2-x-4}$

(b) $\frac{x^2-x-12}{x^2+8x+15} \cdot \frac{x^2+2x-15}{16-x^2}$

(c) $\frac{2x^2+7x-4}{8x^3-4x^2} \cdot \frac{12x^2-24x}{x^2+6x+8}$

(d) $\frac{x^2-7x-8}{1-x^2} \cdot \frac{3x^2-4x+1}{9x^2-1}$

3. When $\frac{24x^{10}}{2y}$ is divided by $\frac{36x^2}{6y^8}$ the result is

(1) $2x^8y^7$

(2) $\frac{3x^5}{2y^7}$

(3) $\frac{x^8}{3y^7}$

(4) $\frac{x^4}{2y^7}$

4. Express the result of each division problem below in simplest form.

(a) $\frac{5x^3-10x^2}{10x^2+40x} \div \frac{x^2-5x+6}{x^2+x-12}$

(b) $\frac{24-18x}{9x^2-16} \div \frac{2x^2+2x}{3x^2+7x+4}$

(c) $\frac{x^2-6x+8}{3x^4-6x^3} \div \frac{4x^2-1}{2x^3-x^2}$

(d) $\frac{49-x^2}{x^2-9x+14} \div \frac{x^2+2x-35}{6-3x}$

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10.8 Problem Set – Combining Rational Expressions with Addition and Subtraction

FLUENCY

1. Combine each of the following using addition. Simply you result whenever possible.

(a) $\frac{3x-1}{6}+\frac{2x+5}{9}$ (b) $\frac{x}{10}+\frac{1}{15x}$ (c) $\frac{3}{7x}+\frac{5}{14x^2}$

2. Combine and simplify each of the following. Note that each pair of fractions already has a common denominator.

(a) $\frac{3x+7}{x+2}+\frac{2x+3}{x+2}$ (b) $\frac{5x+2}{4x-12}-\frac{3x+8}{4x-12}$ (c) $\frac{6x^2-8x}{x^2-25}-\frac{4x^2+2x}{x^2-25}$

3. Combine each of the following using addition. Simplify your final answers.

(a) $\frac{x}{5x+25}+\frac{2x-3}{x^2-3x-40}$ (b) $\frac{x-4}{x^2-24x+128}+\frac{2}{x^2-12x+32}$

4. Which of the following represents the sum of $\frac{1}{x+1}$ and $\frac{1}{x-1}$?

(1) $\frac{2x}{x^2-1}$ (3) $\frac{2}{x-1}$

(2) $\frac{1}{x}$ (4) $\frac{2x}{x^2+1}$

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5. When the expressions $\frac{x^2-8x}{9-x^2}$ and $\frac{3x+6}{9-x^2}$ are added the result can be written as

(1) $\frac{x-5}{x-3}$ (3) $\frac{2-x}{x+3}$

(2) $\frac{x+2}{x-3}$ (4) $\frac{x+7}{x-3}$

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6. Express each of the following differences in simplest form.

(a) $\frac{x+2}{x^2+4x-32}-\frac{4}{x^2-16}$

(b) $\frac{2x+3}{8x^2+6x+1}-\frac{3}{2x^2-x-1}$

7. When $\frac{7x+14}{3x+12}$ is subtracted from $\frac{2x-6}{3x+12}$ the result can be simplified to

(1) $-\frac{5}{3}$ (3) $\frac{10}{3}$

(2) $-\frac{2}{3}$ (4) $\frac{7}{3}$

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10.9 Problem Set – Complex Fractions

Fluency

1. Simplify each of the following numerical complex fractions.

(a) $\dfrac{\frac{1}{4}+\frac{3}{20}}{\frac{1}{2}}$ (b) $\dfrac{\frac{5}{18}+\frac{1}{6}}{\frac{1}{3}}$ (c) $\dfrac{\frac{3}{4}-\frac{1}{5}}{\frac{1}{4}}$

2. Simplify each of the following complex fractions.

(a) $\dfrac{\frac{1}{2}+\frac{1}{3x}}{\frac{3}{10}+\frac{1}{5x}}$ (b) $\dfrac{2-\frac{1}{2x}}{1+\frac{5}{x}}$ (c) $\dfrac{\frac{1}{8}-\frac{1}{2x}}{\frac{1}{12x}-\frac{1}{3x^2}}$

3. Simplify each of the following complex fractions.

(a) $\dfrac{\frac{5}{3x}-\frac{5}{x^2}}{\frac{1}{3}-\frac{3}{x^2}}$ (b) $\dfrac{\frac{x}{10}-\frac{1}{10}-\frac{2}{x}}{\frac{1}{2}-\frac{x}{10}}$ (c) $\dfrac{3-\frac{3}{4x}}{2-\frac{1}{8x^2}}$

4. Simplify each of the following complex fractions.

(a) $\dfrac{\dfrac{x}{x-4}+\dfrac{4}{x-10}}{\dfrac{5x+10}{x^2-14x+40}}$

(b) $\dfrac{\dfrac{3x+2}{x-1}-\dfrac{8}{x-4}}{\dfrac{2x^2-12x}{x^2-5x+4}}$

5. Which of the following is equivalent to $\dfrac{\dfrac{1}{x-1}-\dfrac{1}{x}}{\dfrac{1}{x^2-x}}$?

(1) 1

(2) $\dfrac{2}{x-1}$

(3) $\dfrac{x}{x-1}$

(4) $x-x^2$

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REASONING

6. Since one can multiply by the number 1 at any point in an expression, simplify the following complex fraction by simplifying the more minor complex fraction first, then continue

$$\dfrac{\dfrac{1}{2}-\dfrac{1}{x}}{\dfrac{1}{10x}-\dfrac{1}{\dfrac{\dfrac{x}{2}-1}{\dfrac{1}{10x}-\dfrac{1}{5x^2}}}}$$

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10.10 Problem Set – Polynomial Long Division

FLUENCY

1. Write each of the following rational expressions in the form $a+\frac{r}{x-b}$. Do these by rewriting your numerator as was done in Exercises #4 and #5.

(a) $\frac{x+6}{x+2}$

(b) $\frac{x-10}{x-3}$

(c) $\frac{2x+5}{x+2}$

(d) $\frac{5x-2}{x-4}$

2. If the expression $\frac{10x+11}{2x+1}$ was placed in the form $5+\frac{a}{2x+1}$, then which of the following would be the value of a?

(1) 6

(3) 3

(2) −7

(4) −5

____

3. Use polynomial long division to simplify each of the following ratios. There should be a zero remainder.

(a) $\frac{x^2+5x-24}{x-3}$

(b) $\frac{6x^2+11x-10}{3x-2}$

4. Use polynomial long division to write each of the following ratios in $q(x)+\frac{r}{x-a}$ form, where $q(x)$ is a polynomial and r is the remainder.

(a) $\frac{x^2-6x+11}{x-4}$

(b) $\frac{x^2+2x-25}{x+7}$

(c) $\frac{3x^2+17x+25}{x+4}$

(d) $\frac{5x^2-41x+3}{x-8}$

5. Write each of the following in $q(x)+\frac{r}{x-a}$. The polynomial $q(x)$ will now be a quadratic.

(a) $\frac{x^3+7x^2+17x+41}{x+5}$

(b) $\frac{2x^3-11x^2+22x-25}{x-3}$

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Algebra 2R Unit 9: Complex Numbers

10.11 Problem Set – The Remainder Theorem

FLUENCY

1. Which of the following is the remainder when the polynomial x^2-5x+3 is divided by $(x-8)$?

(1) 107 (3) 3

(2) 27 (4) 9

____

2. If the ratio $\dfrac{2x^2+17x+42}{x+5}$ is placed in the form $q(x)+\dfrac{r}{x+5}$, where $q(x)$ is a polynomial, then which of the following is the correct value of r?

(1) –3 (3) 18

(2) 177 (4) 7

____

3. When the polynomial $p(x)$ was divided by the factor $x-7$ the result was $x+\dfrac{11}{x-7}$. Which of the following is the value of $p(7)$?

(1) –8 (3) 11

(2) 7 (4) It does not exist

____

4. Which of the following binomials is a factor of the quadratic $4x^2-35x+24$? Try to do this without factoring but by using the Remainder Theorem.

(1) $x+6$ (3) $x-8$

(2) $x-4$ (4) $x+2$

____

5. Which of the following linear expressions is a factor of the cubic polynomial $x^3+9x^2+16x-12$?

(1) $x+6$ (3) $x-3$

(2) $x-1$ (4) $x+2$

____

6. Consider the cubic polynomial $p(x)=x^3+x^2-46x+80$.

(a) Using polynomial long division, write the ratio of $\frac{p(x)}{x-3}$ in **quotient-remainder form**, i.e. in the form $q(x)+\frac{r}{x-3}$. Evaluate $p(3)$. How does this help you check your quotient-remainder form?

(b) Evaluate $p(5)$. What does this tell you about the binomial $x-5$?

(c) If $q(x)=\frac{p(x)}{x-5}$, then use polynomial long division to find an expression for the polynomial $q(x)$.

(d) Use your answer from (c) to **completely factor** the cubic polynomial $p(x)$. Besides $x=5$, what are its other zeroes?

7. For the cubic $x^3+7x^2+13x+3$ has only one rational zero, $x=-3$. Use polynomial long division to show that the remainder is zero when dividing the cubic by $x+3$. Then use the quadratic formula to find the other two (irrational) zeroes.

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Algebra 2R Unit 9: Complex Numbers

10.12 Problem Set – Solving Rational Equations

FLUENCY

1. Solve each of the following fractional equations. After "clearing" the denominators you should have a linear equation to solve.

(a) $\frac{x-2}{3}+\frac{x+1}{6}=\frac{3}{2}$

(b) $\frac{13}{2x}-\frac{4}{15}=\frac{31}{6x}$

(c) $\frac{5}{x+2}+\frac{1}{2}=3$

2. Solve each of the fractional equations for all value(s) of x.

(a) $x-8=-\frac{12}{x}$

(b) $\frac{3}{4}+\frac{1}{2x}=\frac{1}{2x}+\frac{1}{3x^2}$

(c) $\frac{17}{x}-\frac{11}{x+3}=\frac{5x+8}{x+3}$

(d) $\frac{x+10}{2}-\frac{13}{x+1}=\frac{11}{3}$

3. Solve the following equation for all values of x. Express your answers in simplest $a+bi$ form.

$$\frac{x}{9}=\frac{x-3}{x-1}$$

4. Solve the following equation for all values of x. Be sure to check for extraneous roots.

$$\frac{x}{\sqrt{x+11}}-1=\frac{1}{\sqrt{x+11}}$$

5. Solve each of the following equations. Be sure to check for extraneous roots.

(a) $\frac{x+1}{x-5}+\frac{2}{x-6}=\frac{2}{x^2-11x+30}$

(b) $\frac{x-3}{x-7}-\frac{1}{x}=\frac{28}{x^2-7x}$

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Algebra 2R Unit 9: Complex Numbers

10.13 Problem Set – Solving Rational Inequalities

FLUENCY

Solve each of the following rational inequalities. Show your answers using a number line and an appropriate notation.

1. $\frac{x-10}{x+5} \geq 0$

2. $\frac{2x+1}{x+3} < 0$

3. $\frac{x^2-4}{x^2-x-20} > 0$

4. $\frac{x^2-6x-16}{x^2-x-6} \leq 0$

5. $\frac{x^2+6x+9}{4x^2-3x-1} \geq 0$

6. $\frac{x^2-12x+36}{4x^2-4x+1} < 0$

For problems 7 through 9, solve each rational inequality by first comparing it to zero. Represent your answers on a number line and using appropriate notation.

7. $\dfrac{x+1}{x-3} \leq 2$

8. $\dfrac{x^2+2x}{x+4} > \dfrac{4}{3}$

9. $\dfrac{1}{x-2} - \dfrac{1}{x+2} \geq \dfrac{3}{x^2-4}$

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Algebra 2R Unit 9: Complex Numbers

10.14 Problem Set – Reasoning About Radical and Rational Equations

FLUENCY

1. Solve the following equation involving a square root. Be sure to reject the extraneous solution (and there will be one).

$$x=-\sqrt{3x+10}$$

2. Solve the following rational equation. Reject any extraneous roots.

$$\frac{x}{x-3}+\frac{2}{x^2-7x+12}=\frac{2}{x-4}$$

REASONING

3. Consider the square root equation $\sqrt{x}=x-2$.

(a) Show that $x=4$ is a solution to this equation.

(b) The value $x=1$ is not a solution to the original equation. Show that after squaring both sides, $x=1$ is a solution to this new equation.

4. Given the equation $2x-1=7$ answer the following.

(a) Solve this equation for the one and only value of x that is a solution.

(b) What extraneous root is introduced if the first step taken to solve the equation is squaring both sides? Show the work that leads to this extraneous root.

5. Consider the equation $-4x=12$, for which $x=-3$ is the only solution.

(a) If Dakota begins to solve the problem in the following way, what property could Dakota use to justify the unusual move of multiplying both sides by the expression $(x-6)$?

$$-4x(x-6)=12(x-6)$$
$$-4x^2+24x=12x-72$$
$$0=4x^2-12x-72$$
$$0=x^2-3x-18$$

(b) Solve the equation $0=x^2-3x-18$. What extraneous root was introduced by multiplying by $(x-6)$ on both sides?

6. Squaring both sides of an equation is **irreversible.** Is cubing both sides of an equation **reversible?** Provide numerical examples to help support your answer.