Name: __________   Course #: ______

A2R

UNIT 4: Exponential and Logarithmic Functions

FLIP NOTES – Part 2

Information in parenthesis is ixl.com reference for practice on the topic.

4.1: INTEGER EXPONENTS

***Exercise* #1:** Given the exponential function $f(x) = 20(2)^x$ evaluate each of the following without using your calculator. Show the calculations that lead to your final answer.

(a) $f(2)$

You Try It!

(a) $f(0)$

(b) $f(-2)$

***Exercise* #2:** For each of the following, write the product as a single exponential expression. Write (a) and (b) as extended products first (if necessary).

(a) $2^3 \cdot 2^4$

You try It!

(a) $2^6 \cdot 2^2$

(b) $2^m \cdot 2^n$

***Exercise* #3:** For each of the following, write the exponential expression in the form 3^x. Write (a) and (b) as extended products first (if necessary).

(a) $\left(3^2\right)^3$

You Try It!

(a) $\left(3^4\right)^2$

(b) $\left(3^m\right)^n$

4.2: RATIONAL EXPONENTS

UNIT FRACTION EXPONENTS

For n given as a positive integer: $b^{1/n} = \sqrt[n]{b}$

***Exercise* #1:** Rewrite each of the following using roots instead of fractional exponents. Then evaluate

(a) $125^{1/3}$

(b) $9^{-1/2}$

You Try It!!

(a) $16^{1/4}$

(b) $32^{-1/5}$

RATIONAL EXPONENT CONNECTION TO ROOTS

For the rational number m/n we define $b^{m/n}$ to be: $\sqrt[n]{b^m}$ or $\left(\sqrt[n]{b}\right)^m$.

***Exercise* #2:** Evaluate each of the following exponential expressions involving rational exponents without the use of your calculator. Show your work. Then, check your final answers with the calculator.

(a) $16^{3/4}$

(b) $8^{-2/3}$

You Try It!

(a) $25^{3/2}$

(b) $27^{2/3}$

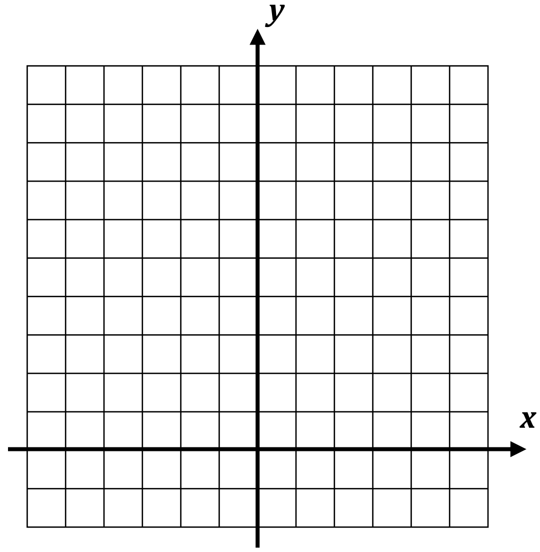
4.3: EXPONENTIAL FUNCTION BASICS

BASIC EXPONENTIAL FUNCTIONS

$y = b^x$ where $b > 0$ and $b \neq 1$

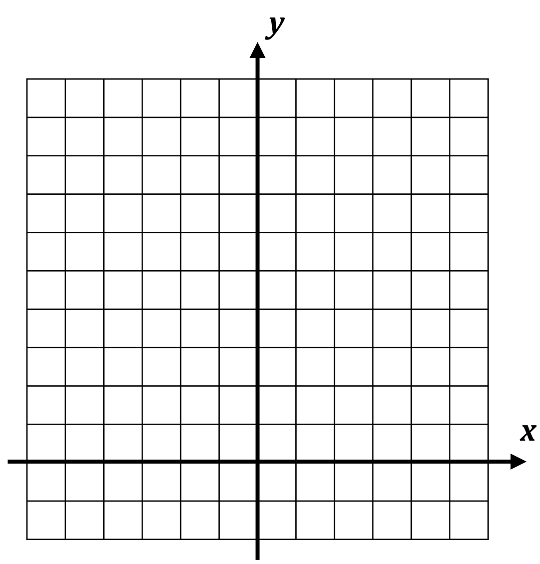
Exercise #1: Consider the function $y = 2^x$. Fill in the table below without using your calculator and then sketch the graph on the grid provided.

x	$y = 2^x$
–3	
–2	
–1	
0	
1	
2	
3	



***You Try It!*:** Now consider the function $y = \left(\frac{1}{2}\right)^x$. Using your calculator to help you, fill out the table below and sketch the graph on the axes provided.

x	$y = \left(\frac{1}{2}\right)^x$
–3	
–2	
–1	
0	
1	
2	
3	

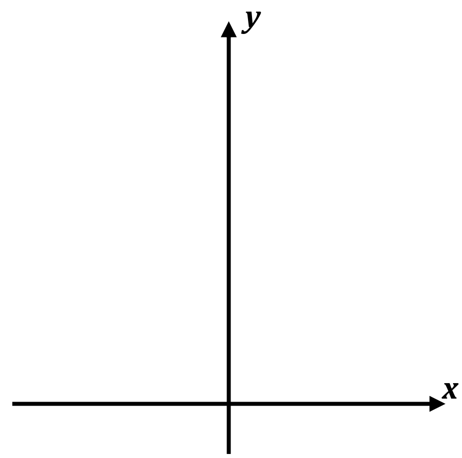


***Exercise* #2:** Now consider the function $y = 7(3)^x$.

(a) Determine the *y*-intercept of this function algebraically. Justify your answer.

(b) Does the exponential function increase or decrease? Explain your choice.

(c) Create a rough sketch of this function, labeling its *y*-intercept.

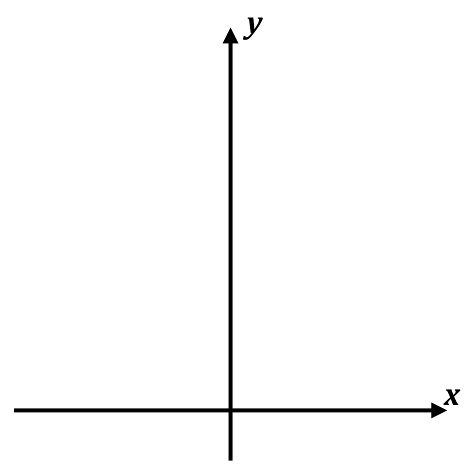


***You Try It*:** Consider the function $y = \left(\frac{1}{3}\right)^x + 4$.

(a) Determine this graph's *y*-intercept algebraically. Justify your answer.

(b) Does the function increase or decrease? Explain your choice.

(c) Create a rough sketch of this function, labeling its *y*-intercept.



4.4: FINDING EQUATIONS OF EXPONENTIAL FUNCTIONS

***Exercise* #1:** For an exponential function of the form $f(x)=a(b)^x$, it is known that $f(0)=8$ and $f(3)=1000$. Find the equation of the exponential function.

You try It! For an exponential function of the form $g(x)=a(b)^x$, it is known that $g(0)=7$ and $g(2)=63$. Find the equation of the exponential function.

***Exercise* #2:** An exponential function of the form $y=a(b)^x$ passes through the points $(2,36)$ and $(5,121.5)$. Find the equation

You Try It! An exponential function of the form $y=a(b)^x$ passes through the points $(2,20)$ and $(4,80)$. Find the equation

4.5: THE METHOD OF COMMON BASES

***Exercise* #1:** Solve each of the following simple exponential equations by writing each side of the equation using **a common base**.

(a) $2^x = 16$

(b) $5^x = \frac{1}{25}$

You Try It!

(a) $3^x = 27$

(b) $16^x = 4$

***Exercise* #2:** Solve each of the following equations by finding a common base for each side.

(a) $8^x = 32$

(b) $125^x = \left(\frac{1}{25}\right)^{4-x}$

You Try It!

(a) $9^{2x+1} = 27$

4.6: Exponential Modeling with Percent Growth and Decay

Increasing Exponential Models

If quantity Q is known to increase by a fixed percentage p, in decimal form, then Q can be modeled by

$$Q(t) = Q_0(1+p)^t$$

where Q_0 represents the amount of Q present at $t = 0$ and t represents time.

***Exercise* #1:** Suppose that you deposit money into a savings account that receives 5% interest per year on the amount of money that is in the account for that year. Assume that you deposit $400 into the account initially.

(a) How much will the savings account increase by over the course of the year?

(b) How much money is in the account at the end of the year?

(c) By what single number could you have multiplied the $400 by in order to calculate your answer in part (b)?

(d) Using your answer from part (c), determine the amount of money in the account after 2 and 10 years. Round all answers to the nearest cent when needed.

(e) Give an equation for the amount in the savings account $S(t)$ as a function of the number of years since the $400 was invested.

(f) Using a table on your calculator determine, to the nearest year, how long it will take for the initial investment of $400 to double. Provide evidence to support your answer.

***You Try It!*:** If the population of a town is decreasing by 4% per year and started with 12,500 residents, what is its projected population in 10 years? Show the exponential model you use to solve this problem. (Hint – decreasing means negative rate)

4.7: MINDFUL MANIPULATION OF PERCENTS

***Exercise* #1:** A population of wombats is growing at a constant percent rate. If the population on January 1st is 1027 and a year later is 1079, what is its yearly percent growth rate to the nearest *tenth* of a percent?

***Exercise* #2:** Now let's try to determine what the percent growth in wombat population will be over a decade of time. We will assume that the rounded percent increase found in *Exercise* #1 continues for the next decade.

(a) After 10 years, what will we have multiplied the original population by, rounded to the nearest hundredth? Show the calculation.

(b) Using your answer from (a), what is the decade percent growth rate?

***Exercise* #3:** Let's stick with our wombats from Exercise #1. Assuming their growth rate is constant over time, what is their monthly growth rate to the nearest tenth of a percent? Assume a constant sized month.

***Exercise* #4:** If a population was growing at a constant rate of 22% every 5 years, then what is its percent growth rate over at 2 year time span? Round to the nearest tenth of a percent.

(a) First, give an expression that will calculate the single year (or yearly) percent growth rate based on the fact that the population grew 22% in 5 years.

(b) Now use this expression to calculate the percent growth over 2 years.

***Exercise* #5:** World oil reserves (the amount of oil unused in the ground) are depleting at a constant 2% per year. We would like to determine what the percent decline will be over the next 20 years based on this 2% yearly decline.

(a) Write and evaluate an expression for what we would multiply the initial amount of oil by after 20 years.

(b) Use your answer to (a) to determine the percent decline after 20 years. Be careful! Round to the nearest percent.

***Exercise* #6:** A radioactive substance's half-life is the amount of time needed for half (or 50%) of the substance to decay. Let's say we have a radioactive substance with a half-life of 20 years.

(a) What percent of the substance would be radioactive after 40 years?

(b) What percent of the substance would be radioactive after only 10 years? Round to the nearest tenth of a percent.

(c) What percent of the substance would be radioactive after only 5 years? Round to the nearest tenth of a percent.

4.8: Introduction to Logarithms

***Exercise* #1:** For each of the below logarithms. (a) Write an equivalent exponential equation (b) Evaluate without a calculator.

(a) $\log_2 8$ (b) $\log_4 16$ (c) $\log_5 625$ (d) $\log_{10} 100,000$

(e) $\log_6 \left(\frac{1}{36} \right)$ (f) $\log_2 \left(\frac{1}{16} \right)$ (g) $\log_5 \sqrt{5}$ (h) $\log_3 \sqrt[5]{9}$

You Try It!

If the function $y = \log_2 (x+8)+9$ was graphed in the coordinate plane, what would be the y-intercept?

Common Logarithms

Exercise #2: Evaluate without calculator, then verify with your calculator

(a) $\log 100$ (b) $\log \left(\frac{1}{1000} \right)$

You try It!

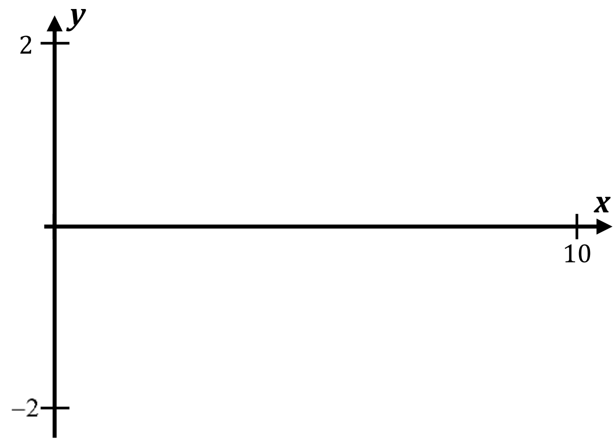
(a) $\log 1000$ (b) $\log \sqrt{10}$ (hint: write as a rational exponent)

4.9: GRAPHS OF LOGARITHMS

***Exercise* #1:** Using your calculator, sketch the graph of $y = \log_{10} x$ on the axes below. Label the x-intercept. State the domain and range of $y = \log_{10} x$.

Domain:

Range:



DOMAIN AND RANGE OF LOGS

You Try It!

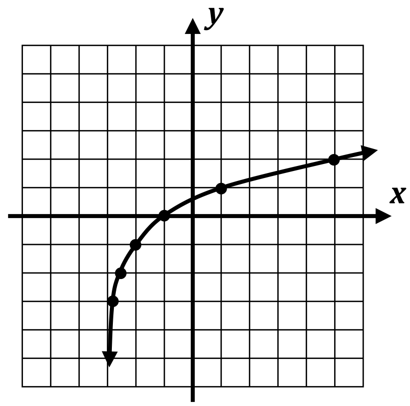
Which of the following equations describes the graph shown below? Show or explain how you made your choice.

(1) $y = \log_3(x+2) - 1$

(2) $y = \log_2(x-3) + 1$

(3) $y = \log_2(x+3) - 1$

(4) $y = \log_3(x+3) - 1$



4.10: LOGARITHM LAWS
EXPONENT AND LOGARITHM LAWS

LAW	EXPONENT VERSION	LOGARITHM VERSION
Product	$b^x \cdot b^y = b^{x+y}$	$\log_b(x \cdot y) = \log_b x + \log_b y$
Quotient	$\frac{b^x}{b^y} = b^{x-y}$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
Power	$(b^x)^y = b^{x \cdot y}$	$\log_b(x^y) = y \cdot \log_b x$

Example 1: Use the properties of logs to expand $\log_4 6x^5$

You Try It!

Use the properties of logs to expand $\log_7\left(\frac{3y^4}{x^3}\right)$

Example 2: Use the properties of logarithms to write as a single logarithm $\log_2(10) + 3\log_2 x$

You Try It!

Write as a single logarithm 3ln2 – (1/2)ln64

4.11: SOLVING EXPONENTIAL EQUATIONS USING LOGARITHMS

***Example* #1:** Solve: $4^x = 8$ using (a) common bases and (b) the logarithm laws

(a) Method of Common Bases

(b) Logarithm Approach

Example #2: Solve each of the following equation for the value of *x*. Round your answers to the nearest *hundredth*.

$5^x = 18$

You Try It!

$4^x = 100$

***Example* #3:** A biologist is modeling the population of bats on a tropical island. When he first starts observing them, there are 104 bats. The biologist believes that the bat population is growing at a rate of 3% per year.

(a) Write an equation for the number of bats, $B(t)$, as a function of the number of years, *t*, since the biologist started observing them.

(b) Using your equation from part (a), algebraically determine the number of years it will take for the bat population to reach 200. Round your answer to the nearest year.

You try It!

A stock has been declining in price at a steady pace of 5% per week. If the stock started at a price of $22.50 per share, determine algebraically the number of weeks it will take for the price to reach $10.00. Round your answer to the nearest week

4.12: THE NUMBER e AND THE NATURAL LOGARITHM

THE NUMBER e

1. Like π, e is irrational.
2. $e \approx 2.72$
3. Used in Exponential Modeling

Example 1: A population of llamas on a tropical island can be modeled by the equation $P = 500e^{0.035t}$, where t represents the number of years since the llamas were first introduced to the island.

(a) How many llamas were initially introduced at $t = 0$? Show the calculation that leads to your answer.

(b) Algebraically determine the number of years for the population to reach 600. Round your answer to the nearest *tenth* of a year.

You Try It!

A small town in Italy, Tozzoa, is inhabited by devilishly handsome, insanely intelligent people whose numbers are dwindling due to various frustration induced illnesses. The population can be modeled by the equation P = $5200e^{-0.025t}$

a. What is the initial population and rate of decay?

b. Algebraically determine after how many years will there be only 2600 Tozzoans left?

THE NATURAL LOGARITHM

The inverse of $y = e^x$: $y = \ln x \quad (y = \log_e x)$

Example 2: Evaluate the below without a calculator. You can leave your answer in terms of *e*

a. lm1

b. $\ln e^4$

c. $\ln \sqrt{e}$

You Try It!

a. $\ln e$

b. $e^{\ln e^5}$

4.13: COMPOUND INTEREST

Example 1**:** How much would \$1000 invested at a nominal 2% yearly rate, compounded monthly, be worth in 20 years? Show the calculations that lead to your answer.

You Try It!
How much would \$1000 invested at a nominal rate of 5%, compounded ***weekly***, be worth in 20 years? Show the calculations that lead to your answer.

CONTINUOUS COMPOUND INTEREST

For an initial principal, P, compounded continuously at a nominal yearly rate of r, the investment would be worth an amount A given by:

$$A(t) = Pe^{rt}$$

Example 2**:** A person invests \$350 in a bank account that promises a nominal rate of 2% continuously compounded.

(a) Write an equation for the amount this investment would be worth after t-years.

(b) How much would the investment be worth after 20 years?

(c) Algebraically determine the time it will take for the investment to reach \$400. Round to the nearest tenth of a year.

(d) What is the effective annual rate for this investment? Round to the nearest hundredth of a percent.

You Try it!
Michael invested \$500 in a bank account that promises a nominal rate of 3% continuously compounded. How much will he have in 10 years?