Name: ____________________ Course: ___

A2R

UNIT 2: FUNCTIONS AS THE CORNERSTONE OF ALGEBRA 2

PROBLEM SETS

- 2.1: Introduction to Functions
- 2.2: Function Notation
- 2.3 Function Composition
- 2.4: The Domain and Range of a Function
- 2.5: One-to-One Functions
- 2.6: Inverse Functions
- 2.7: Key Features of Functions

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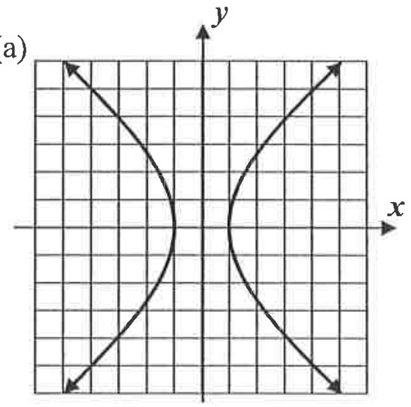
Algebra 2R Unit 2: Functions as the Cornerstones of Algebra 2

2.1 Problem Set – Introduction to Functions

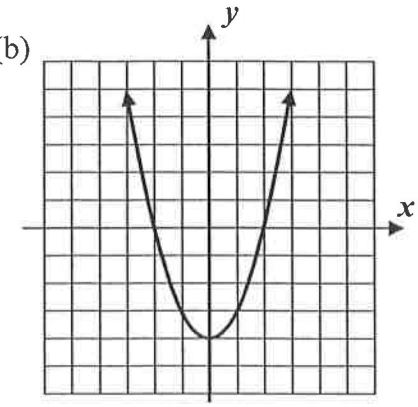
FLUENCY

1. Determine for each of the following graphed relationships whether y is a function of x using the Vertical Line Test.

(a)



(b)



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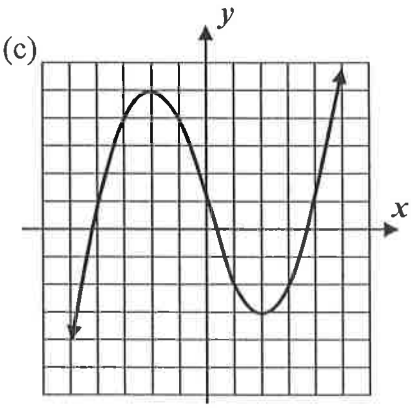

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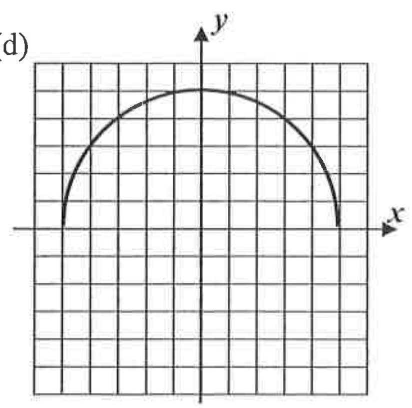

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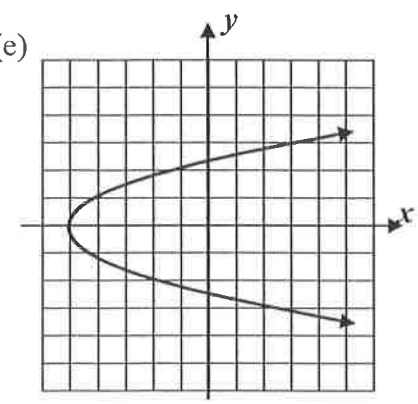
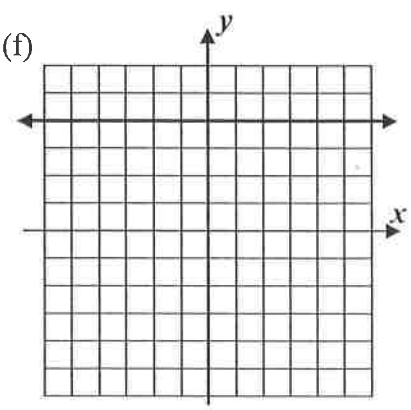


(f)



2. What are the outputs for an input of $x = 5$ given functions defined by the following formulas:

(a) $y = 3x - 4$ (b) $y = 50 - 2x^2$ (c) $y = 2^x$

APPLICATIONS

3. Evin is walking home from the museum. She starts 38 blocks from home and walks 2 blocks each minu Evin's distance from home is a function of the number of minutes she has been walking.

   (a) Which variable is independent and which variable is dependent in this scenario?

   (b) Fill in the table below for a variety of time values.

Time, t, in minutes	0	1	5	10
Distance from home, D, in blocks				

   (c) Determine an equation relating the distance, D, that Evin is from home as a function of the number of minutes, t, that she has been walking.

   (d) Determine the number of minutes, t, that it takes for Evin to reach home.

REASONING

4. In one of the following tables, the variable y is a function of the variable x. Explain which relationship is a function and why the other is not.

x	y
-2	11
0	7
2	11
4	23
6	43

Relationship #1

x	y
0	0
1	-1
1	1
4	-2
4	2

Relationship #2

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Algebra 2R Unit 2: Functions as the Cornerstones of Algebra 2

2.2 Problem Set – Function Notation

FLUENCY

1. Without using your calculator, evaluate each of the following given the function definitions and input values.

(a) $f(x)=3x+7$

$f(-4)=$

$f(2)=$

(b) $g(x)=3x^2$

$g(2)=$

$g(-3)=$

(c) $h(x)=\sqrt{x-5}$

$h(41)=$

$h(14)=$

2. Using **STORE** on your calculator, evaluate each of the following more complex functions.

(a) $f(x)=\dfrac{3x^2-5}{4x+10}$

$f(-5)=$

$f(0)=$

(b) $g(x)=\dfrac{\sqrt{25-x^2}}{x}$

$g(4)=$

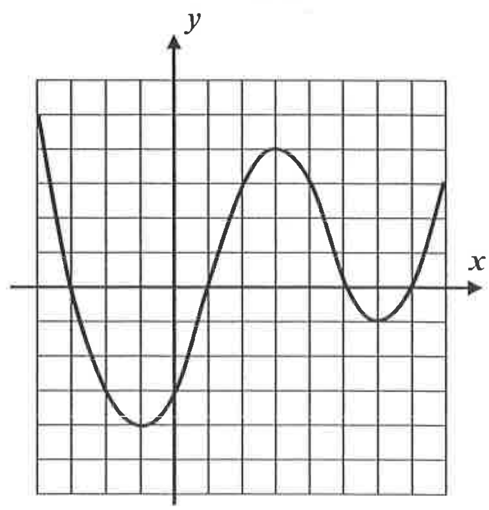
$g(-3)=$

(c) $h(x)=30(1.2)^x$

$h(3)=$

$h(0)=$

3. Based on the graph of the function $y=g(x)$ shown below, answer the following questions.



(a) Evaluate $g(-2)$, $g(0)$, $g(3)$ and $g(7)$.

(b) What values of x solve the equation $g(x)=0$

(c) Graph the horizontal line $y=2$ on the grid above and label.

(d) How many values of x solve the equation $g(x)=2$?

APPLICATIONS

4. Ian invested \$2500 in an investment vehicle that is guaranteed to earn 4% interest compounded yearly. T amount of money, A, in his account as a function of the number of years, t, since creating the account is given by the equation $A(t)=2500(1.04)^t$.

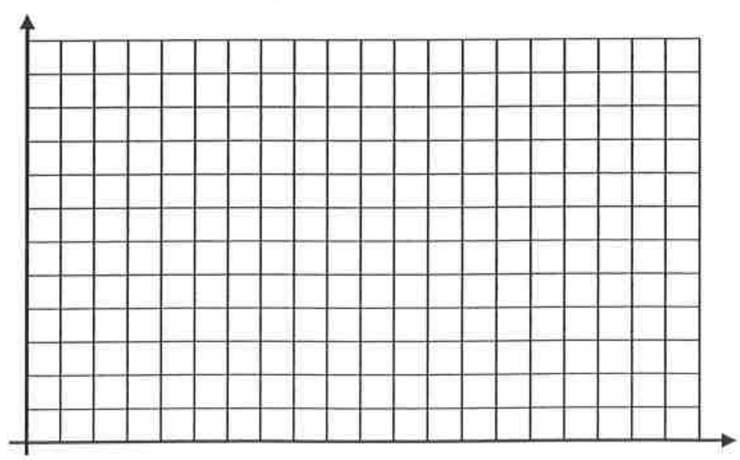
(a) Evaluate $A(0)$ and $A(10)$.

(b) What do the two values that you found in part (a) represent?

(c) Using tables on your calculator, determine, to the nearest whole year, the value of t that solves the equation $A(t)=5000$. Justify your answer with numerical evidence.

(d) What does the value of t that you found in part (b) represent about Ian's investment?

5. A ball is shot from an air-cannon at an angle of 45° with the horizon. It travels along a path given by the equation $h(d)=-\frac{1}{50}d^2+d$, where h represents the ball's height above the ground and d represents the distance the ball has traveled horizontally. Using your calculator to generate a table of values, graph this function for all values of d on the interval $0 \le d \le 50$. Look at the table to properly scale the y-axis.



What is the maximum height that the ball reaches? At what value of d does it reach this height?

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Algebra 2R Unit 2: Functions as the Cornerstones of Algebra 2

2.3 Problem Set – Function Composition

FLUENCY

1. Given $f(x)=3x-4$ and $g(x)=-2x+7$ evaluate:

(a) $f(g(0))$ (b) $g(f(-2))$ (c) $f(f(3))$

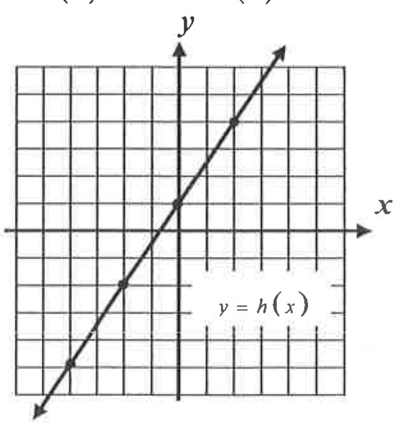
(d) $(g\circ f)(6)$ (e) $(f\circ g)(5)$ (f) $(g\circ g)(2)$

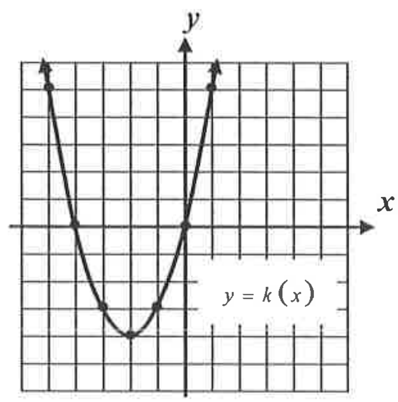
2. Given $h(x)=x^2+11$ and $g(x)=\sqrt{x-2}$ evaluate:

(a) $h(g(18))$ (b) $g(h(4))$ (c) $(g\circ g)(11)$

(d) $h(h(0))$ (e) $(h\circ g)(38)$ (f) $(g\circ h)(0)$

3. The graphs of $y=h(x)$ and $y=k(x)$ are shown below. Evaluate the following based on these two graphs.





(a) $h(k(-2))$ (b) $(k\circ h)(0)$ (c) $h(h(-2))$ (d) $(k\circ k)(-2)$

4. If $g(x)=3x-5$ and $h(x)=2x-4$ then $(g\circ h)(x)=?$

(1) $6x-17$ (3) $5x-9$

(2) $6x-14$ (4) $x-1$

______

5. If $f(x)=x^2+5$ and $g(x)=x+4$ then $f(g(x))=$

(1) x^2+9 (3) $4x^2+20$

(2) $x^2+8x+21$ (4) x^2+21

______

APPLICATIONS

6. Scientists modeled the intensity of the sun, I, as a function of the number of hours *since* 6:00 a.m., h, using the function $I(h)=\frac{12h-h^2}{36}$. They then model the temperature of the soil, T, as a function of the intensity using the function $T(I)=\sqrt{5000I}$. Which of the following is closest to the temperature of the soil at 2:00 p.m. ?

(1) 54 (3) 67

(2) 84 (4) 38

______

7. Physics students are studying the effect of the temperature, T, on the speed of sound, S. They find that the speed of sound in meters per second is a function of the temperature in degrees Kelvin, K, by $S(K)=\sqrt{410K}$. The degrees Kelvin is a function of the temperature in Celsius given by $K(C)=C+273.15$. Find the speed of sound when the temperature is 30 degrees Celsius. Round to the nearest *tenth*.

REASONING

8. Consider the functions $f(x)=2x+9$ and $g(x)=\frac{x-9}{2}$. Calculate the following.

(a) $g(f(15))$ (b) $g(f(-3))$ (c) $g(f(x))$

(d) What appears to always be true when you compose these two functions?

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Algebra 2R Unit 2: Functions as the Cornerstones of Algebra 2

2.4 Problem Set – The Domain and Range of a Function

FLUENCY

1. A function is given by the set of ordered pairs $\{(2,5),(4,9),(6,13),(8,17)\}$. Write its domain and range in roster form.

Domain: Range:

2. The function $h(x)=x^2+5$ maps the domain given by the set $\{-2,-1,0,1,2\}$. Which of the following sets represents the range of $h(x)$?

(1) $\{0,6,10,12\}$ (3) $\{5,6,9\}$

(2) $\{5,6,7\}$ (4) $\{1,4,5,6,9\}$

______

3. Which of the following values of x would *not* be in the domain of the function defined by $f(x)=\dfrac{x-2}{x+3}$?

(1) $x=-3$ (3) $x=3$

(2) $x=2$ (4) $x=-2$

______

4. Determine any values of x that do not lie in the domain of the function $f(x)=\dfrac{3x+2}{2x-10}$. Justify your response.

5. Which of the following values of x *does* lie in the domain of the function defined by $g(x)=\sqrt{2x-7}$?

(1) $x=0$ (3) $x=3$

(2) $x=2$ (4) $x=5$

______

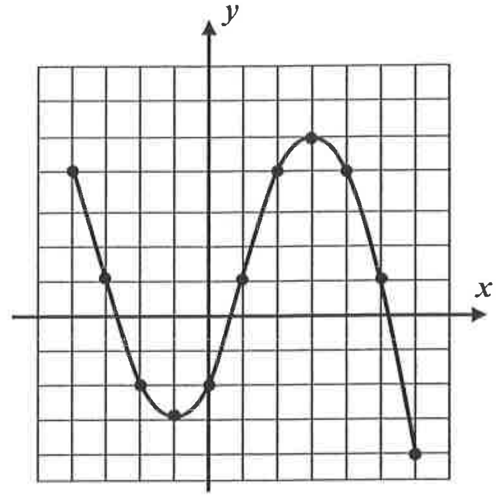
6. Which of the following would represent the domain of the function $y=\sqrt{6-2x}$?

(1) $\{x:x>3\}$ (3) $\{x:x\le 3\}$

(2) $\{x:x<3\}$ (4) $\{x:x\ge 3\}$

______

7. The function $y=f(x)$ is completely defined by the graph shown below.



(a) Evaluate $f(-4), f(3)$, and $f(6)$.

(b) Draw a rectangle that whose vertices are $(-4, 5)$, $(6, 5), (6, -4)$, and $(-4, -4)$.

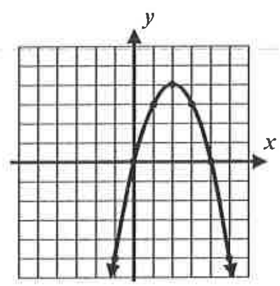
(c) State the domain and range of this function using interval notation.

Domain: Range:

8. Which of the following represents the range of the quadratic function shown in the graph below?

(1) $(4, \infty)$ (3) $(-\infty, 4)$

(2) $(-\infty, 4]$ (4) $[4, \infty)$



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APPLICATIONS

9. A child starts a piggy bank with \$2. Each day, the child receives 25 cents at the end of the day and puts it in the bank. If A represents the amount of money and d stands for the number of days then $A(d)=2+0.25d$ gives the amount of money in the bank as a function of days (think about this formula).

(a) Evaluate $A(1), A(7)$, and $A(30)$.

(b) For what value of d will $A(d)=\$10.50$.

(c) Explain why the domain does not contain the value $d=2.5$.

(d) Explain why the range does not include the value $A=\$3.10$.

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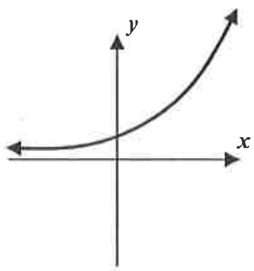
Algebra 2R Unit 2: Functions as the Cornerstones of Algebra 2

2.5 Problem Set – One-To-One Functions

FLUENCY

1. Which of the following graphs illustrates a one-to-one relationship?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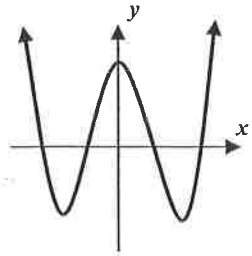
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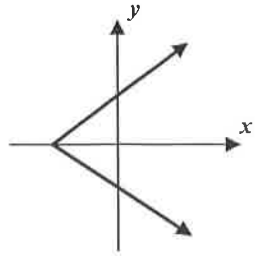
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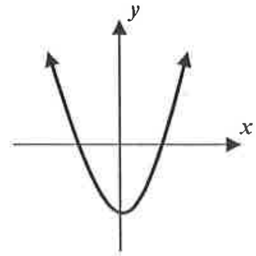
(4)

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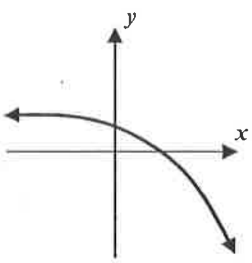
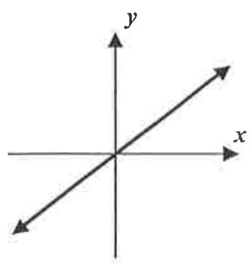
2. Which of the following graphs does *not* represent that of a one-to-one function?


(1)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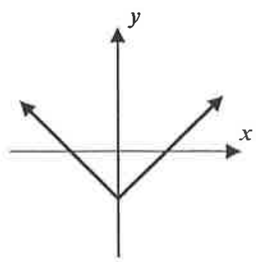
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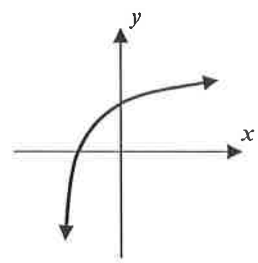
(4)

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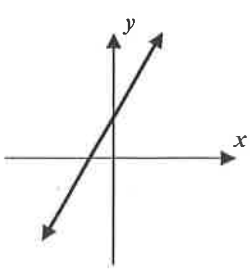
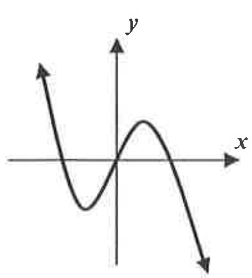
3. In which of the following graphs is each input *not* paired with a *unique* output?


(1)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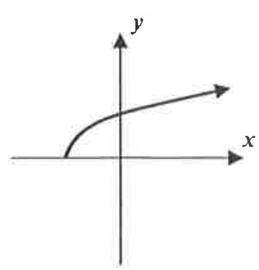
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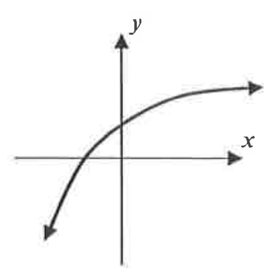
(4)

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4. In which of the following formulas is the variable y a one-to-one function of the variable x? (Hint – try generating some values either in your head or using TABLES on your calculator.)

(1) $y = x^2$ (3) $y = 2x$

(2) $y = |x|$ (4) $y = 5$

____

5. Which of the following tables illustrates a relationship in which y is a one-to-one function of x?

(1)

x	y
-2	-1
0	-3
2	-1
4	1
6	3

(2)

x	y
-2	-8
-1	-1
0	0
1	1
2	8

(3)

x	y
-2	-5
-1	-4
0	-1
-1	7
-2	5

(4)

x	y
-2	11
-1	-4
0	-5
1	-4
2	11

______

APPLICATIONS

6. A recent newspaper gave temperature data for various days of the week in table format. In which of the tables below is the reported temperature a one-to-one function of the day of the week?

(1)

x	y
Mon	75
Tue	68
Wed	65
Thu	74

(2)

x	y
Mon	75
Tue	72
Wed	68
Thu	72

(3)

x	y
Mon	58
Tue	52
Mon	81
Tue	76

(4)

x	y
Mon	56
Tue	58
Mon	85
Tue	85

______

7. Physics students drop a basketball from 5 feet above the ground and its height is measured each tenth of a second until it stops bouncing. The height of the basketball, h, is clearly a function of the time, t, since was dropped.

(a) Sketch the general graph of what you believe this function would look like.

(b) Is the height of the ball a one-to-one function of time? Explain your answer.

REASONING

8. Consider the function $f(x) = \text{round}(x)$, which rounds the input, x, to the nearest integer. Is this function one-to-one? Explain or justify your answer.

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Algebra 2R Unit 2: Functions as the Cornerstones of Algebra 2

2.6 Problem Set -- Inverse Functions

FLUENCY

1. If the point $(-7,5)$ lies on the graph of $y=f(x)$, which of the following points must lie on the graph of its inverse?

(1) $(5,-7)$ (3) $(7,-5)$

(2) $\left(-\frac{1}{7},\frac{1}{5}\right)$ (4) $\left(\frac{1}{7},-\frac{1}{5}\right)$

____

2. The function $y=f(x)$ has an inverse function $y=f^{-1}(x)$. If $f(a)=-b$ then which of the following must be true?

(1) $f^{-1}(-b)=-a$ (3) $f^{-1}(-b)=a$

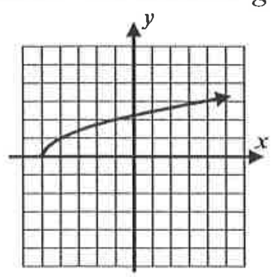
(2) $f^{-1}\left(\frac{1}{a}\right)=-\frac{1}{b}$ (4) $f^{-1}(b)=-a$

____

3. The graph of the function $y=g(x)$ is shown below. The value of $g^{-1}(2)$ is

(1) 2.5 (3) 0.4

(2) -4 (4) -1



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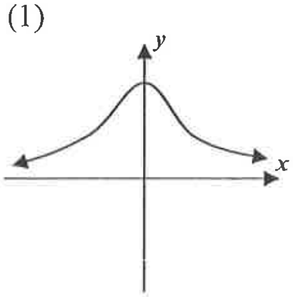
4. Which of the following functions would have an inverse that is also a function?

(1)



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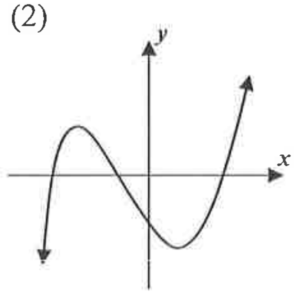
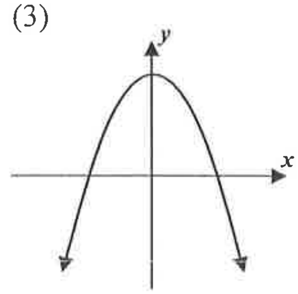


(3)



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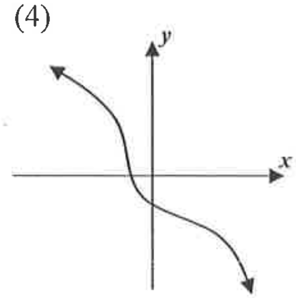

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5. For a one-to-one function it is known that $f(0)=6$ and $f(8)=0$. Which of the following must be true about the graph of this function's inverse?

(1) its y-intercept = 6 (3) its x-intercept = -6

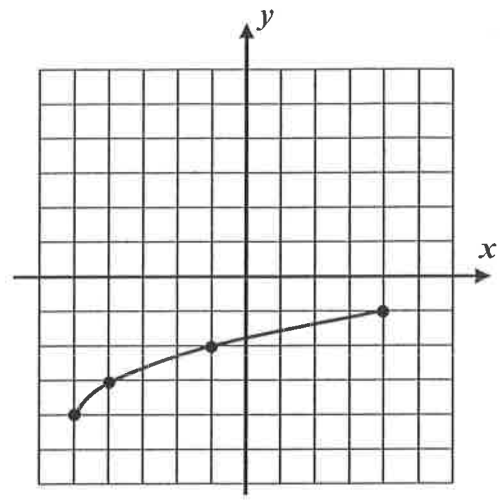
(2) its y-intercept = 8 (4) its x-intercept = -8

____

6. The function $y=h(x)$ is entirely defined by the graph shown below.

(a) Sketch a graph of $y=h^{-1}(x)$. Create a table of values if needed.

(b) Write the domain and range of $y=h(x)$ and $y=h^{-1}(x)$ using interval notation.



$y=h(x)$

Domain:

Range:

$y=h^{-1}(x)$

Domain:

Range:

APPLICATIONS

7. The function $y=A(r)=\pi r^2$ is a one-to-one function that uses a circle's radius as an input and gives the circle's area as its output. Selected values of this function are shown in the table below.

r	1	2	3	4	5	6
$A(r)$	π	4π	9π	16π	25π	36π

(a) Determine the values of $A^{-1}(9\pi)$ and $A^{-1}(36\pi)$ from using the table.

(b) Determine the values of $A^{-1}(100\pi)$ and $A^{-1}(225\pi)$.

(c) The original function $y=A(r)$ converted an input, the circle's radius, to an output, the circle's area. What are the inputs and outputs of the inverse function?

Input:

Output:

REASONING

8. The domain and range of a one-to-one function, $y=f(x)$, are given below in set-builder notation. Give the domain and range of this function's inverse also in set-builder notation.

$y=f(x)$

Domain: $\{x \mid -3 \le x < 5\}$

Range: $\{y \mid y > -2\}$

$y=f^{-1}(x)$

Domain:

Range:

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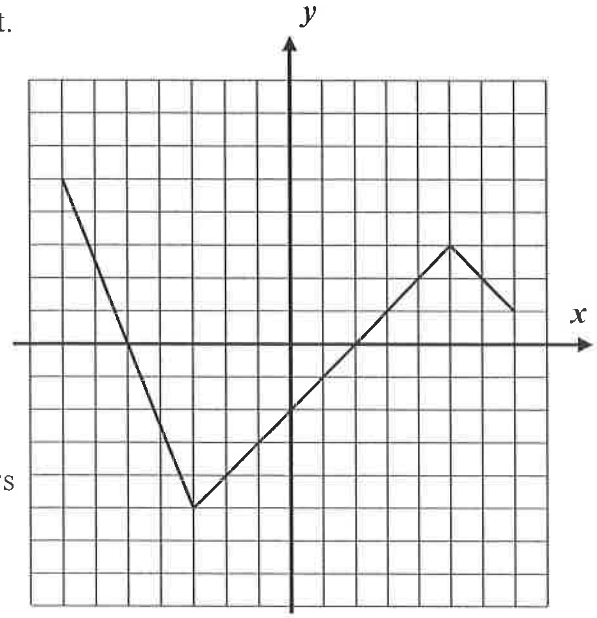
Algebra 2R Unit 2: Functions as the Cornerstones of Algebra 2

2.7 Problem Set – Key Features of Functions

FLUENCY

1. The piecewise linear function $f(x)$ is shown to the right. Answer the following questions based on its graph.



(a) Evaluate each of the following based on the graph:

(i) $f(4)=$ (ii) $f(-3)=$

(b) State the zeroes of $f(x)$.

(c) Over which of the following intervals is $f(x)$ always increasing?

(1) $-7<x<-3$ (3) $-5<x<5$

(2) $-3<x<5$ (4) $-5<x<3$

(d) State the coordinates of the relative maximum and the relative minimum of this function.

Relative Maximum:__________________

Relative Minimum:__________________

(e) Over which of the following intervals is $f(x)<0$?

(1) $-7<x<-3$ (3) $-5<x<2$

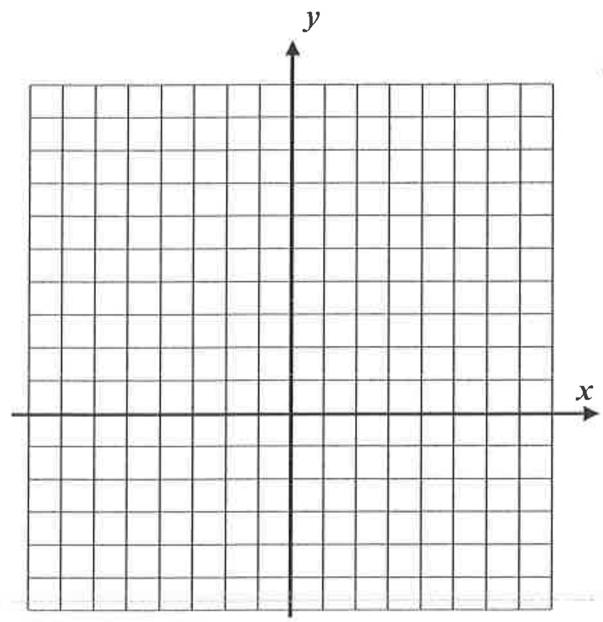
(2) $2\le x\le 7$ (4) $-5\le x\le 2$

(f) A second function $g(x)$ is defined using the rule $g(x)=2f(x)+5$. Evaluate $g(0)$ using this rule. What does this correspond to on the graph of g?

(g) A third function $h(x)$ is defined by the formula $h(x)=x^3-3$. What is the value of $g(h(2))$? Show how you arrived at your answer.

2. For the function $g(x) = 9 - (x+1)^2$ do the following.

(a) Sketch the graph of g on the axes provided.



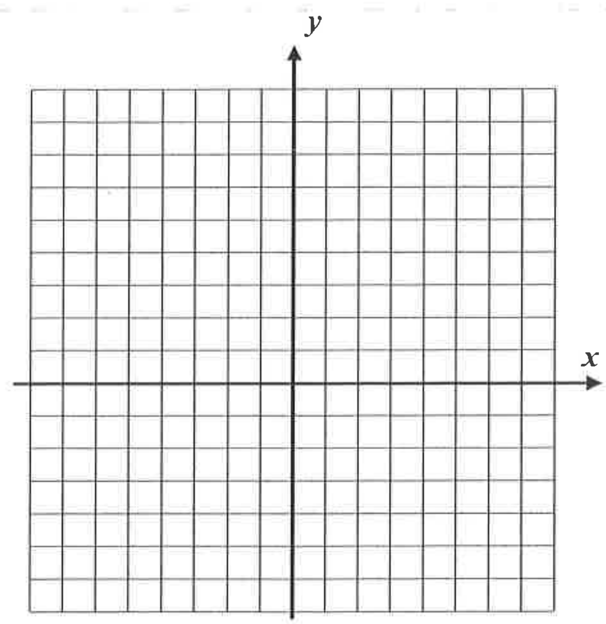
(b) State the zeroes of g.

(c) Over what interval is $g(x)$ decreasing?

(d) Over what interval is $g(x) \geq 0$?

(e) State the range of g.

3. Draw a graph of $y = f(x)$ that matches the following characteristics.



Increasing on: $-8 < x < -4$ and $-1 < x < 5$

Decreasing on: $-4 < x < -1$

$f(-8) = -5$ and zeroes at $x = -6, -2$, and 3

Absolute maximum of 7 and absolute minimum of -5

4. A continuous function has a domain of $-7 \leq x \leq 10$ and has selected values shown in the table below. The function has exactly two zeroes and a relative maximum at $(-4, 12)$ and a relative minimum at $(5, -6)$.

x	-7	-4	-1	0	2	5	7	10
$f(x)$	8	12	0	-2	-5	-6	0	4

(a) State the interval on which $f(x)$ is decreasing.

(b) State the interval over which $f(x) < 0$.