Name: ___________________ **Course:** ___

A2R

UNIT 2: FUNCTIONS AS THE CORNERSTONE OF ALGEBRA 2

CLASS NOTES

- 2.1: Introduction to Functions
- 2.2: Function Notation
- 2.3 Function Composition
- 2.4: The Domain and Range of a Function
- 2.5: One-to-One Functions
- 2.6: Inverse Functions
- 2.7: Key Features of Functions

2.1: INTRODUCTION TO FUNCTIONS

DEFINITION: A **function** is any "rule" that assigns exactly one output value (y-value) for each input value (x-value). These rules can be expressed in different ways, the most common being equations, graphs, and tables of values. We call the input variable **independent** and output variable **dependent**.

***Exercise* #1:** An internet music service offers a plan whereby users pay a flat monthly fee of $5 and can then download songs for 10 cents each.

(a) What are the independent and dependent variables in this scenario?

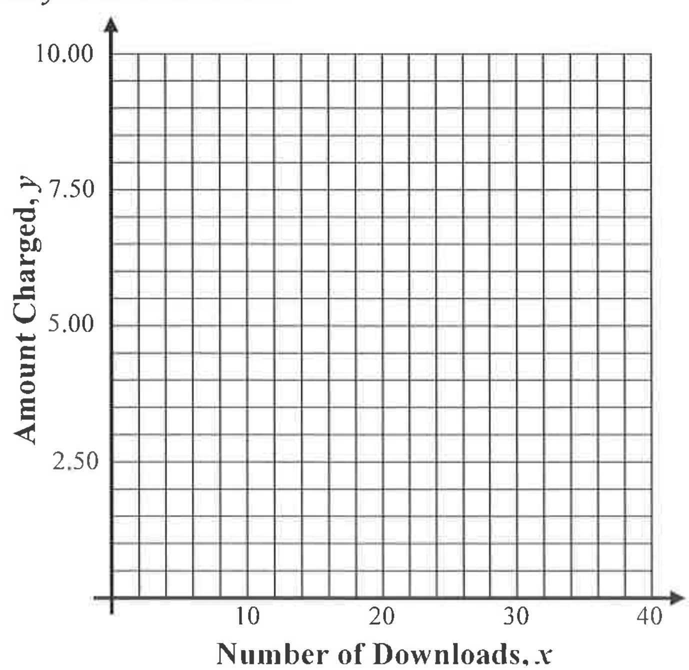
Independent: Dependent:

(b) Fill in the table below for a variety of independent values:

Number of downloads, x	0	5	10	20
Amount Charged, y				

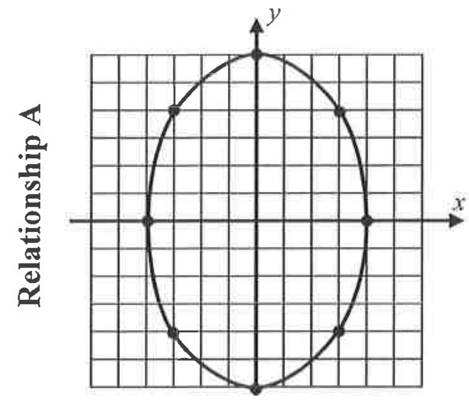
(c) Let the number of downloads be represented by the variable x and the amount charged in dollars be represented by the variable y, write an equation that models y as a function of x.

(d) Based on the equation you found in part (c), produce a graph of this function for all values of x on the interval $0 \le x \le 40$. Use a calculator **TABLE** to generate additional coordinate pairs to the ones you found in part (b).



***Exercise* #2:** One of the following graphs shows a relationship where y is a function of x and one does not.

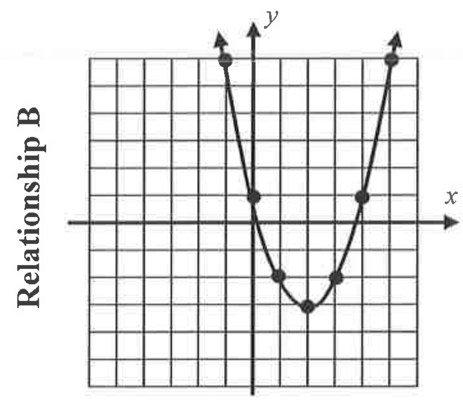
(a) Draw the vertical line whose equation is $x = 3$ on both graphs.



(b) Give all output values for each graph at an input of 3 .

Relationship A: Relationship B:

(c) Explain which of these relationships is a function and why.



2.2: FUNCTION NOTATION

***Exercise* #1:** Evaluate each of the following given the function definitions and input values.

(a) $g(x) = x^2 + 4$

$g(0) =$

(c) $h(x) = 2^x$

$h(-2) =$

FUNCTION NOTATION

$$y = f\ (x)$$

Output ↑ Rule ↑ Input

***Exercise* #2:** Boiling water at 212 degrees Fahrenheit is left in a room that is at 65 degrees Fahrenheit and begins to cool. Temperature readings are taken each hour and are given in the table below. In this scenario, the temperature, *T*, is a function of the number of hours, *h*.

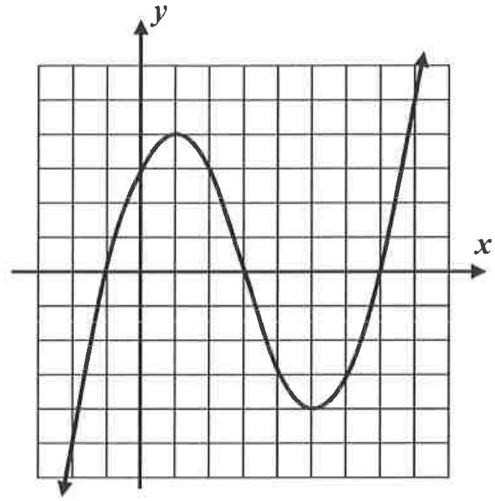
h (hours)	0	1	2	3	4	5	6	7	8
$T(h)$ $(^\circ F)$	212	141	104	85	76	70	68	66	65

(a) Evaluate $T(2)$ and $T(6)$.

(b) For what value of *h* is $T(h) = 76$?

(c) Between what two consecutive hours will $T(h) = 100$?

***Exercise* #3:** The function $y = f(x)$ is defined by the graph shown below. Answer the following questions based on this graph.



(a) Evaluate $f(-1)$, $f(1)$, and $f(5)$.

(b) Evaluate $f(0)$. What special feature on a graph does $f(0)$ always correspond to?

(c) What values of x solve the equation $f(x) = 0$? What special features on a graph does the set of x-values that solve $f(x) = 0$ correspond to?

(d) Between what two consecutive integers does the largest solution to $f(x) = 3$ lie?

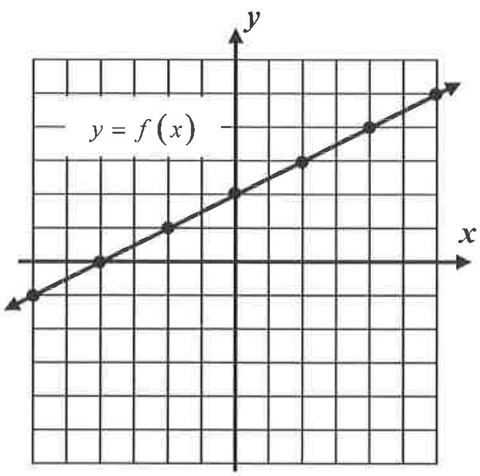
2.3: FUNCTION COMPOSITION

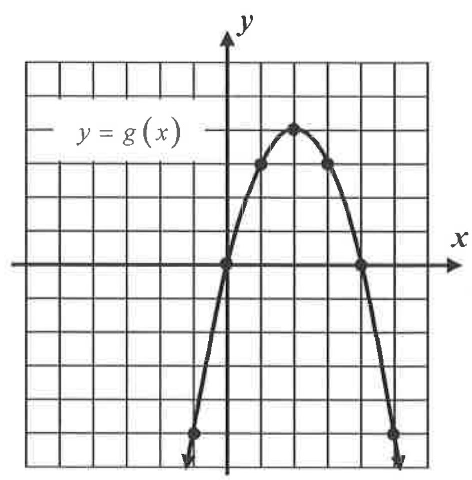
***Exercise* #1:** Given $f(x) = x^2 - 5$ and $g(x) = 2x + 3$, find values for each of the following.

(a) $f(g(1)) =$ (b) $g(f(2)) =$ (c) $g(g(0)) =$

(d) $(f \circ g)(-2) =$ (e) $(g \circ f)(3) =$ (f) $(f \circ f)(-1) =$

***Exercise* #2:** The graphs below are of the functions $y = f(x)$ and $y = g(x)$. Evaluate each of the following questions based on these two graphs.





(a) $g(f(2)) =$ (b) $f(g(-1)) =$ (c) $g(g(1)) =$

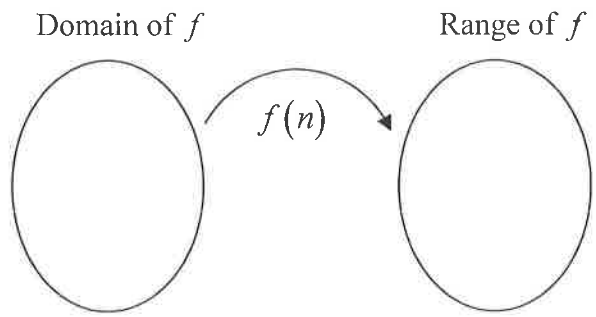
(d) $(g \circ f)(-2) =$ (e) $(f \circ g)(0) =$ (f) $(f \circ f)(0) =$

.***Exercise* #3:** Given the functions $f(x) = 3x - 2$ and $g(x) = 5x + 4$, determine formulas in simplest $y = ax + b$ form for:

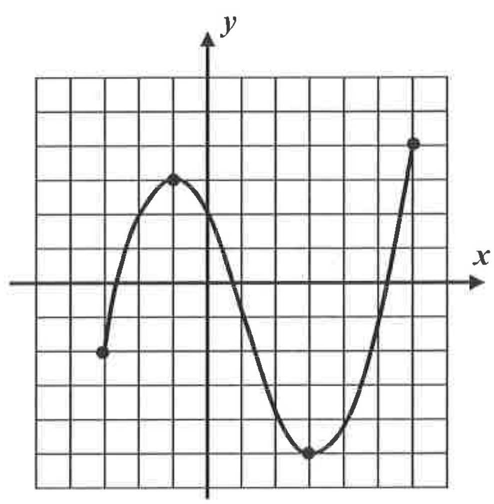
(a) $f(g(x))$ (b) $g(f(x))$

2.4: THE DOMAIN AND RANGE OF A FUNCTION

***Exercise* #1:** State the range of the function $f(n)=2n+1$ if its domain is the set $\{1, 3, 5\}$. Show the domain and range in the mapping diagram below.



***Exercise* #2:** The function $y=g(x)$ is completely defined by the graph shown below. Answer the following questions based on this graph.



(a) Determine the minimum and maximum x-values represented on this graph.

(b) Determine the minimum and maximum y-values represented on this graph.

(c) State the domain and range of this function using set builder notation.

***Exercise* #4:** The function $f(x)=\dfrac{2x+1}{x-4}$ has outputs given by the following calculator table.

(a) Evaluate $f(1)$ and $f(6)$ from the table.

(b) Why does the calculator give an ERROR at $x=4$?

(c) Are there any values except $x=4$ that are not in the domain of f? Explain.

x	$f(x)$
1	-1
2	-2.5
3	-7
4	Error
5	11
6	6.5
7	5

***Exercise* #5:** Which of the following values of x would not be in the domain of the function $y=\sqrt{x+4}$? Explain your answer.

(1) $x=0$

(2) $x=5$

(3) $x=-3$

(4) $x=-8$

2.5: ONE-TO-ONE FUNCTIONS

ONE-TO-ONE FUNCTIONS

A function $f(x)$ is called one-to-one if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$.
(In other words, different inputs give different outputs.)

***Exercise* #1:** Of the four tables below, one represents a relationship where y is a one-to-one function of x. Determine which it is and explain why the others are not.

(1)

x	y
4	2
4	-2
9	3
9	-3

(2)

x	y
-2	1
-1	0
0	1
1	2

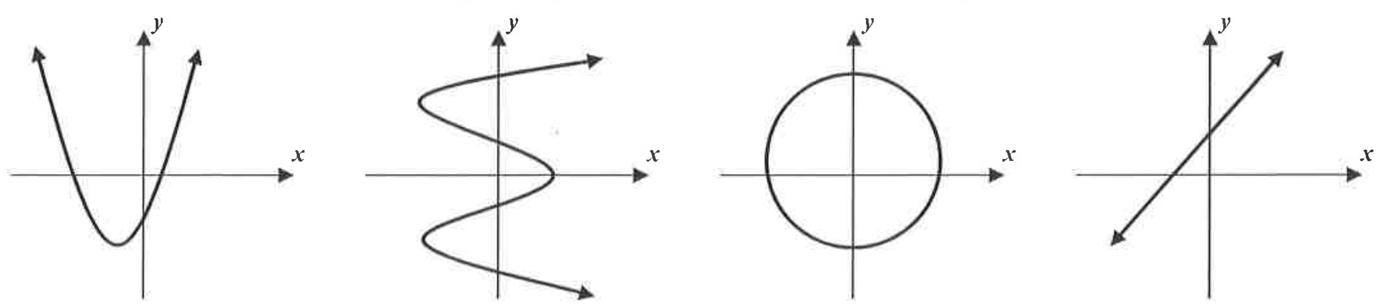
(3)

x	y
1	2
2	4
3	8
4	16

(4)

x	y
-3	10
-2	9
-1	7
-2	10

***Exercise* #2:** Consider the following four graphs which show a relationship between the variables y and x.



(1) (2) (3) (4)

(a) Circle the two graphs above that are functions. Explain how you know they are functions.

(b) Of the two graphs you circled, which is one-to-one? Explain how you can tell from its graph.

THE HORIZONTAL LINE TEST

If any given horizontal line passes through the graph of a function at most one time, then that function is one-to-one. This test works because horizontal lines represent constant y-values; hence, if a horizontal line intersects a graph more than once, an output has been repeated.

2.6: INVERSE FUNCTIONS

***Exercise* #1:** Consider the two linear functions given by the formulas $f(x)=\frac{3x+7}{2}$ and $g(x)=\frac{2x-7}{3}$.

(a) Calculate $f(5)$ and $g(11)$.

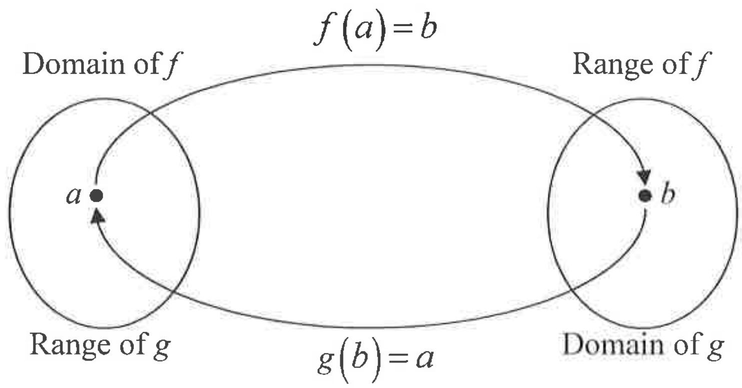
(b) Calculate $f(0)$ and $g\left(\frac{7}{2}\right)$.

(c) Calculate $f(g(-1))$.

(d) Calculate $f(g(5))$.

(e) Without calculation, determine the value of $f(g(\pi))$.

The two functions seen in Exercise #1 are inverses because they literally "undo" one another. The general idea of inverses, $f(x)$ and $g(x)$, is shown below in the mapping diagram.



***Exercise* #2:** If the point $(-3, 5)$ lies on the graph of $y=f(x)$, then which of the following points must lie on the graph of its inverse?

(1) $(3,-5)$

(2) $(-5,3)$

(3) $(5,-3)$

(4) $\left(-\frac{1}{3},\frac{1}{5}\right)$

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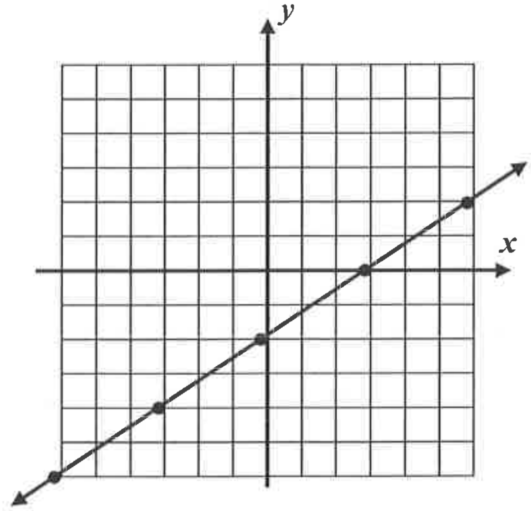
INVERSE FUNCTION NOTATION

If a function $y=f(x)$ has an inverse that is also a function we represent it as $y=f^{-1}(x)$.

.***Exercise* #3:** The linear function $f(x) = \frac{2}{3}x - 2$ is shown graphed below. Use its graph to answer the following questions.



(a) Evaluate $f^{-1}(2)$ and $f^{-1}(-4)$.

(b) Determine the y-intercept of $f^{-1}(x)$.

(c) On the same set of axes, draw a graph of $y = f^{-1}(x)$.

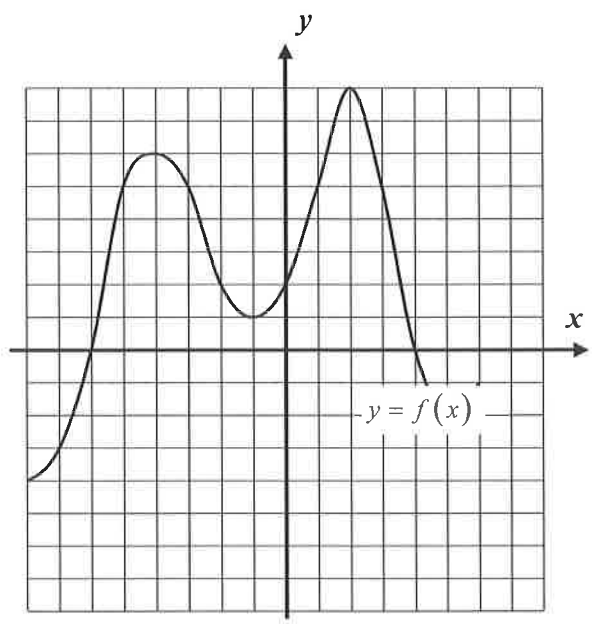
EXISTENCE OF INVERSE FUNCTIONS

A function will have an inverse that is also a function if and only if it is one-to-one. Hence, a quick way to know if a function has an inverse that is also a function is to apply the Horizontal Line Test.

2.7: KEY FEATURES OF FUNCTIONS

The graphs of functions have many key features whose terminology we will be using all year. It is important to master this terminology, most of which you learned in Common Core Algebra I.

***Exercise* #1:** The function $y = f(x)$ is shown graphed to the right. Answer the following questions based on this graph.



(a) State the y-intercept of the function.

(b) State the x-intercepts of the function. What is the alternative name that we give the x-intercepts?

(c) Over the interval $-1 < x < 2$ is $f(x)$ increasing or decreasing? How can you tell?

(d) Give the interval over which $f(x) > 0$. What is a quick way of seeing this visually?

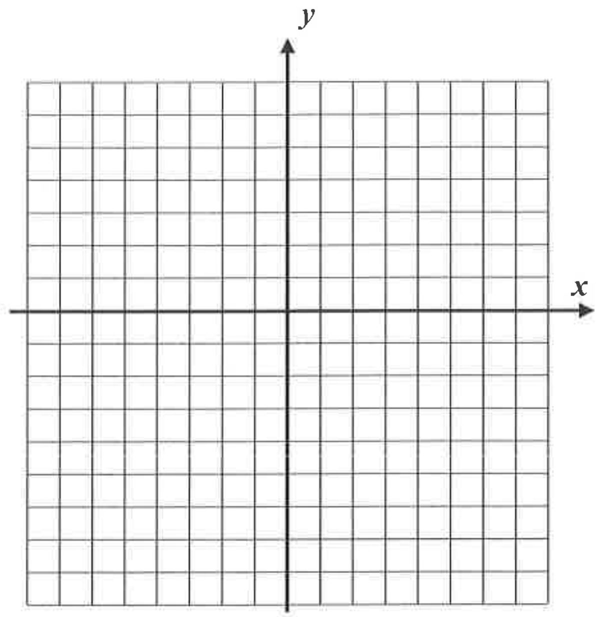
(e) State all the x-coordinates of the relative maximums and relative minimums. Label each.

(f) What are the absolute maximum and minimum values of the function? Where do they occur?

(g) State the domain and range of $f(x)$ using interval notation.

(h) If a second function $g(x)$ is defined by the formula $g(x) = \frac{1}{2} f(x+2)$, then what is the y-intercept of g?

***Exercise* #2:** Consider the function $g(x)=2|x-1|-8$ defined over the domain $-4\le x\le 7$.



(a) Sketch a graph of the function to the right.

(b) State the domain interval over which this function is decreasing.

(c) State zeroes of the function on this interval.

(d) State the interval over which $g(x)\le 0$

(e) Evaluate $g(0)$ by using the algebraic definition of the function. What point does this correspond to on the graph?

(f) Are there any relative maximums or minimums on the graph? If so, which and what are their coordinates?

***Exercise* #3:** A **continuous** function $f(x)$ has a domain of $-6\le x\le 13$ with selected values shown below. The function has exactly two zeroes and has exactly two turning points, one at $(3,-4)$ and one at $(9,3)$.

x	-6	-1	0	3	5	8	9	13
$f(x)$	5	0	-2	-4	-1	0	3	1

(a) State the interval over which $f(x)<0$.

(b) State the interval over which $f(x)$ is increasing.