A2R

UNIT 11: THE CIRCULAR FUNCTIONS

PROBLEM SETS

Information in parenthesis is ixl.com reference for practice.

Name: ______________________________ Course: _______ Date: __________________

Algebra 2R Unit 11: The Circular Functions

11.1 Problem Set – Rotations and Angle Terminology

FLUENCY

1. For each of the following angles, draw a rotation diagram and then state the quadrant the terminal ray of the angles falls within.

(a) $\theta = 135^\circ$ (b) $\theta = 300^\circ$ (c) $\theta = -110^\circ$

(d) $\theta = -310^\circ$ (e) $\theta = 85^\circ$ (f) $\theta = 560^\circ$

2. For each of the following angles, draw a rotation diagram and determine the reference angle.

(a) $\alpha = 245^\circ$ (b) $\alpha = 290^\circ$ (c) $\alpha = 130^\circ$

(d) $\alpha = -242^\circ$ (e) $\alpha = 475^\circ$ (f) $\alpha = -432^\circ$

3. Give two angles that are coterminal with each of the following angles. Make one of the coterminal angles positive and one negative.

(a) $\theta = 105^\circ$ (b) $\theta = 220^\circ$ (c) $\theta = 80^\circ$ (d) $\theta = -245^\circ$

4. When drawn in standard position, which of the following angles is coterminal to one that measures 130°?

(1) 430° (3) 850°

(2) -70° (4) 730°

5. Which of the following angles, when drawn in standard position, would *not* be coterminal with an angle that measures 270°?

(1) -90° (3) 630°

(2) 990° (4) 720°

6. Which of the following angles would *not* have a reference angle equal to 30°?

(1) 210° (3) 120°

(2) -330° (4) -30°

REASONING

7. Angles are a measurement of rotation about a point. Are two coterminal angles the same rotation? Explain your answer. Diagrams are helpful.

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11.2 Problem Set – Radian Angle Measurement

FLUENCY

1. Convert each of the following common degree angles to angles in radians. Express your answers in exact terms of pi.

(a) 30° (b) 45° (c) 60° (d) 180°

(e) 300° (f) 135° (g) 270° (h) 330°

2. Convert each of the following angles given in radians into an equivalent measure in degrees. Your answers will be integers.

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{2}$ (c) $\frac{11\pi}{4}$ (d) $-\frac{4\pi}{3}$

3. If an angle is drawn in standard position with each of the following radians angles, determine the quadrant its terminal ray lies in. Hint – convert each angle into degrees.

(a) 4.75 (b) –5.28 (c) 1.65 (d) 7.38

4. Draw a rotation diagram for each of the following radian angles, which are expressed in terms of pi. Then, determine the reference angle for each, also in terms of pi. Think back to how you did this with degrees.

(a) $\frac{2\pi}{3}$ (b) $\frac{11\pi}{6}$ (c) $\frac{5\pi}{4}$

APPLICATIONS

5. A dog is attached to a 10 foot leash. He travels around an arc that has a length of 25 feet. Which of the following represents the radian angle he has rotated through?

(1) 5 (3) 2.5

(2) 7.5π (4) 1.25π ________

6. A wheel whose diameter is 3 feet rolls a distance of 45 feet without slipping. Through what radian angle did the wheel rotate?

(1) 30 (3) 30π

(2) 25 (4) 12π ________

7. The distance from the center of a Ferris wheel to a person who is riding is 38 feet. What distance does a person travel if the Ferris wheel rotates through an angle of 4.25 radians?

(1) 80.75 feet (3) 507 feet

(2) 42.5 feet (4) 161.5 feet

____

8. A golfer swings a club about a pivot point. If the head of the club travels a distance of 26 feet and rotates through an angle of 5 radians, which of the following gives the distance the club head is from the pivot point?

(1) 1.7 feet (3) 5.2 feet

(2) 2.6 feet (4) 7.2 feet

____

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11.3 Problem Set – The Unit Circle

FLUENCY

1. Draw a rotation diagram for each of the following angles and then determine the ordered pair that lies on the unit circle for each angle.

(a) $\theta = 330^\circ$ (b) $\theta = 135^\circ$ (c) $\theta = -270^\circ$

(d) $\theta = -240^\circ$ (e) $\theta = 540^\circ$ (f) $\theta = -300^\circ$

2. Draw a rotation diagram for each of the following radian angles and then determine the ordered pair that lies on the unit circle for each angle.

(a) $\alpha = \frac{2\pi}{3}$ (b) $\alpha = -\frac{3\pi}{2}$ (c) $\alpha = \frac{11\pi}{6}$

(d) $\alpha = -\frac{\pi}{2}$ (e) $\alpha = \frac{3\pi}{4}$ (f) $\alpha = \frac{4\pi}{3}$

3. All of the points on the unit circle must satisfy the equation $x^2+y^2=1$. Verify that this equation is true for each of the coordinate points given below.

(a) $(-1,0)$

(b) $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

(c) $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$

4. There are other points on the unit circle besides the ones that we determined in this lesson. Every point, though, must satisfy the equation $x^2+y^2=1$. For each of the following problems, either the x or y coordinate of a point on the unit circle is given. Find all possibilities for the other coordinate for this point using the unit circle equation.

(a) $x=\frac{3}{5}$

(b) $y=-\frac{5}{13}$

(c) $x=\frac{1}{4}$

5. For each of the following angles, determine its reference angle. Then state the coordinate on the unit circle for **both** the angle and its reference. What do you notice about the coordinate pairs?

(a) $\theta=150^{\circ}$

(b) $\theta=225^{\circ}$

(c) $\theta=300^{\circ}$

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11.4 Problem Set – The Definition of the Sine and Cosine Functions

FLUENCY

1. Which of the following is the value of $\sin\left(60^\circ\right)$?

(1) $\frac{\sqrt{2}}{2}$ (3) $\frac{\sqrt{3}}{2}$

(2) $\frac{1}{2}$ (4) $\frac{2}{3}$

____

2. Written in exact form, $\cos\left(135^\circ\right)=?$

(1) $-\frac{1}{2}$ (3) $-\frac{\sqrt{3}}{2}$

(2) $-\frac{\sqrt{2}}{2}$ (4) $-\frac{\pi}{4}$

____

3. Which of the following is not equal to $\sin\left(270^\circ\right)$?

(1) $\cos\left(180^\circ\right)$ (3) $-\sin\left(90^\circ\right)$

(2) $-\cos\left(0^\circ\right)$ (4) $\sin\left(360^\circ\right)$

____

4. The terminal ray of an angle drawn in standard position passes through the point $(0.28,-0.96)$, which lies on the unit circle. Which of the following represents the sine of this angle?

(1) –0.96 (3) 0.28

(2) –0.68 (4) –0.29

____

5. The point $A(-5,12)$ lies on the circle whose equation is $x^2+y^2=169$. Which of the following would represent the cosine of an angle drawn in standard position whose terminal rays passes through A?

(1) –5 (3) $-\frac{5}{13}$

(2) $-\frac{5}{12}$ (4) 12

____

6. Which of the following values cannot be the sine of an angle? Hint, think about the range of y-values on the unit circle.

(1) $\frac{7}{13}$ (3) $-\frac{3}{2}$

(2) $-\frac{\sqrt{5}}{3}$ (4) $\frac{\sqrt{11}}{4}$ ______

7. For an angle drawn in standard position, it is known that its cosine is negative and its sine is positive. The terminal ray of this angle must terminate in which quadrant?

(1) I (3) III

(2) II (4) IV ______

8. If both the sine and cosine of an angle are less than zero, then when drawn in standard position in which quadrant would the terminal ray fall?

(1) I (3) III

(2) II (4) IV ______

9. Which of the following has a cosine that is different from $\sin(30^\circ)$?

(1) 60° (3) -60°

(2) -300° (4) 120° ______

10. When drawn in standard position, an angle α has a terminal ray that lies in the second quadrant and whose sine is equal to $\frac{9}{41}$. Find the cosine of α in rational form (as a fraction).

11. If the terminal ray of β lies in the fourth quadrant and $\sin(\beta) = -\frac{\sqrt{3}}{3}$ determine $\cos(\beta)$ in simplest form.

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11.5 Problem Set – More Work with the Sine and Cosine Functions

FLUENCY

1. If $f(x)=10\sin(x)-3$ then $f(30^\circ)=?$

(1) $-\sqrt{3}/2-3$ (3) $-5/2$

(2) 2 (4) $4/3-\sqrt{3}/2$ ________

2. If $f(x)=2x$ and $g(x)=\cos(x)$ then $g\left(f\left(\frac{\pi}{2}\right)\right)=?$

(1) 1 (3) 0

(2) $-\sqrt{2}/2$ (4) -1 ________

3. Which of the following represents a rational number?

(1) $\sin\left(\frac{\pi}{6}\right)$ (3) $\cos\left(\frac{\pi}{4}\right)$

(2) $\sin\left(\frac{2\pi}{3}\right)$ (4) $\cos\left(\frac{5\pi}{4}\right)$ ________

4. When drawn in standard position, an angle β has a terminal ray that lies in the third quadrant. It is known that $\cos(\beta)=-\frac{8}{17}$. Which of the following represents the value of $\sin(\beta)$?

(1) $-\frac{9}{17}$ (3) $-\frac{15}{17}$

(2) $\frac{8}{9}$ (4) $\frac{7}{9}$ ________

5. Which of the following is equal to $\sin(300^\circ)$?

(1) $\sin(60^\circ)$ (3) $-\sin(60^\circ)$

(2) $\sin(30^\circ)$ (4) $-\sin(30^\circ)$ ________

6. For an angle α it is known that its reference angle has a sine value of $\frac{4}{5}$. If the terminal ray of α, when drawn in standard position, falls in the third quadrant then what is the value of $\cos(\alpha)$?

(1) $-\frac{3}{5}$ (3) $-\frac{4}{5}$

(2) $\frac{3}{4}$ (4) $\frac{5}{3}$

______

7. The point $E(-7,-24)$ lies on the circle whose equation is $x^2+y^2=625$. If an angle is drawn in standard position and its terminal ray passes through *E*, what is the value of the sine of this angle?

(1) -7 (3) -24

(2) $-\frac{7}{24}$ (4) $-\frac{24}{25}$

______

8. If it is known that $\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$ and $\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$ then find the value of each of the following. To begin, first determine in which quadrant each angle's terminal ray lies.

(a) $\sin\left(\frac{2\pi}{3}\right)$ (b) $\cos\left(\frac{4\pi}{3}\right)$ (c) $\sin\left(\frac{5\pi}{3}\right)$ (d) $\cos\left(-\frac{2\pi}{3}\right)$

9. Which of the following could not be the value of $\sin(\theta)$? Explain how you can tell.

(1) $-\frac{11}{13}$ (3) $\frac{\sqrt{34}}{5}$

(2) $-\frac{\sqrt{23}}{5}$ (4) $\frac{1}{2}$

______

10. A person on a Ferris wheel sits a distance of 45 feet from the Ferris wheel's center. If they are at an angle of 120°, when measured in standard position, then how high above the center of the wheel are they, to the nearest foot?

(1) 39 feet (3) 23 feet

(2) 12 feet (4) 32 feet

______

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11.6 Problem Set – Basic Graphs of Sine and Cosine

FLUENCY

1. On the grid below, sketch the graphs of each of the following equations based on the basic sine function.

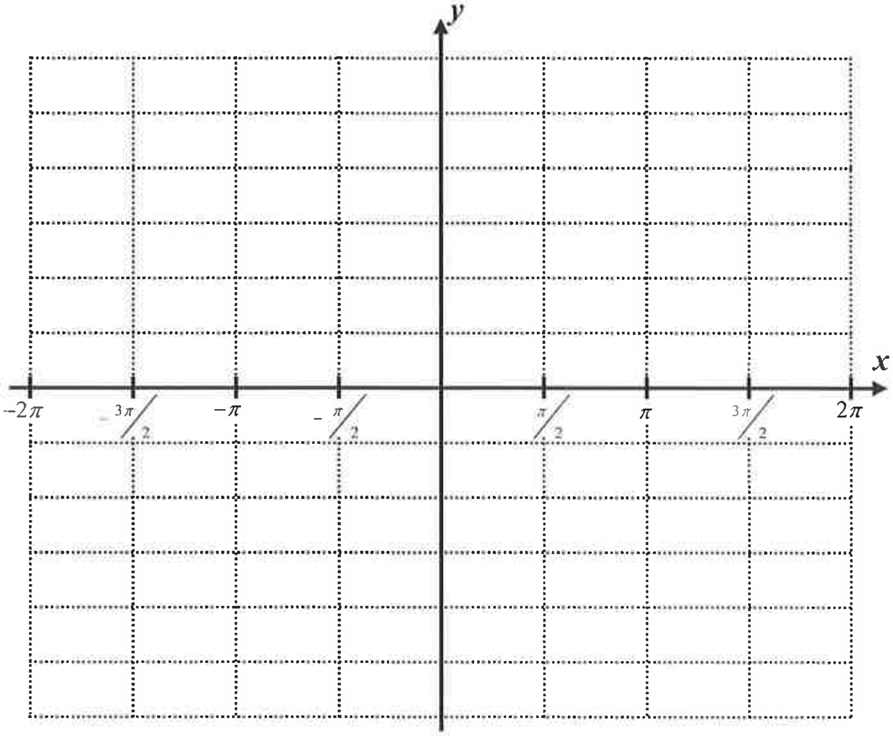
$y=\sin(x)$

$y=3\sin(x)$

$y=-\sin(x)$

$y=-5\sin(x)$

$y=\frac{7}{2}\sin(x)$



2. On the grid below, sketch the graphs of each of the following equations based on the basic cosine function.

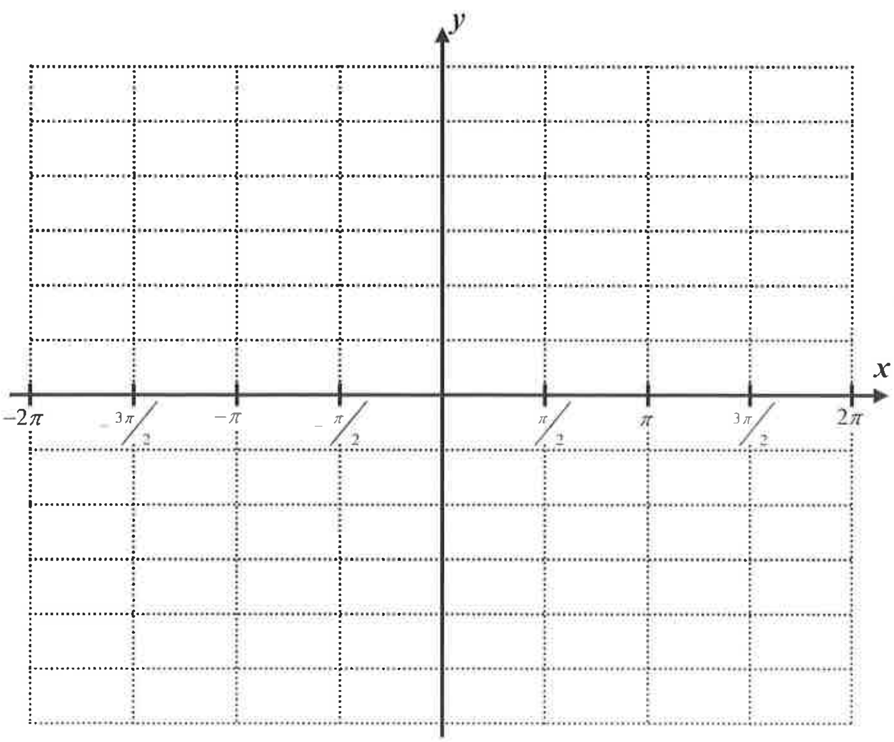
$y=\cos(x)$

$y=4\cos(x)$

$y=-3\cos(x)$

$y=2.5\cos(x)$

$y=-5.5\cos(x)$



3. Which of the following represents the *range* of the trigonometric function $y = 7\sin(x)$?

(1) $(-7, 7)$ (3) $[0, 7)$

(2) $[-7, 7]$ (4) $(-7, 7]$

4. Which of the following is the period of $y = \cos(x)$?

(1) π (3) 2π

(2) 2 (4) $\frac{3\pi}{2}$

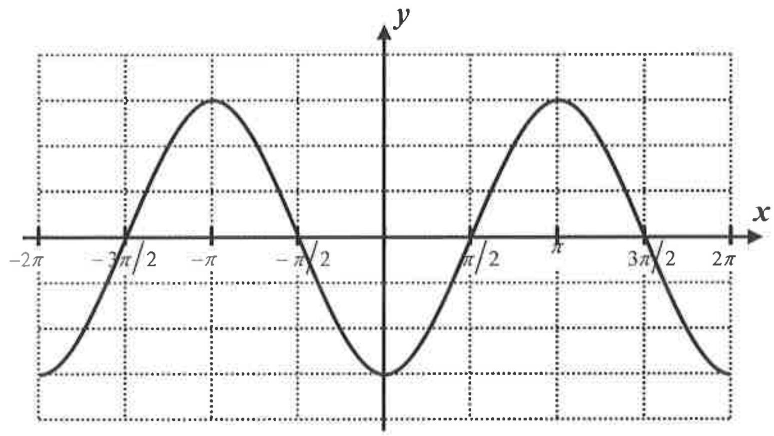
5. Which of the following equations describes the graph shown below?

(1) $y = 3\cos(x)$

(2) $y = -3\cos(x)$

(3) $y = 3\sin(x)$

(4) $y = -3\sin(x)$



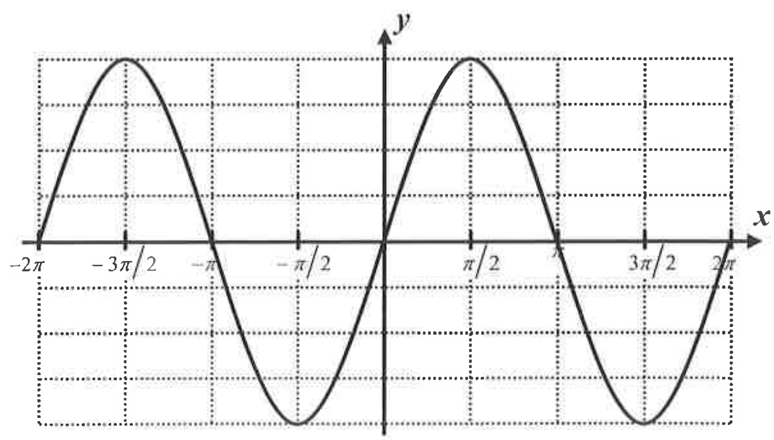
6. Which of the following equations represents the periodic curve shown below?

(1) $y = 4\cos(x)$

(2) $y = -4\cos(x)$

(3) $y = 4\sin(x)$

(4) $y = -4\sin(x)$



7. Which of the following lines when drawn would *not* intersect the graph of $y = 6\sin(x)$?

(1) $x = 8$ (3) $y = -4$

(2) $x = 3$ (4) $y = 9$

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11.7 Problem Set – Vertical Shifting of Sinusoidal Graphs

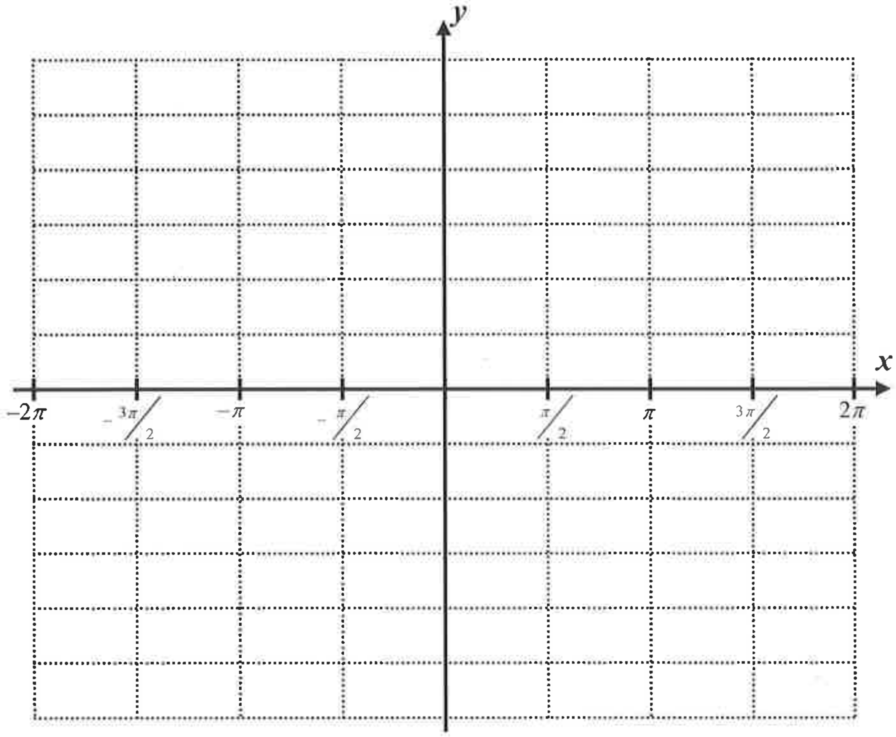
FLUENCY

1. Sketch each of the following equations on the graph grid below. Label each with its equation.

$y = 4\sin(x) + 2$

$y = 2\cos(x) - 4$

$y = -\sin(x) + 4$

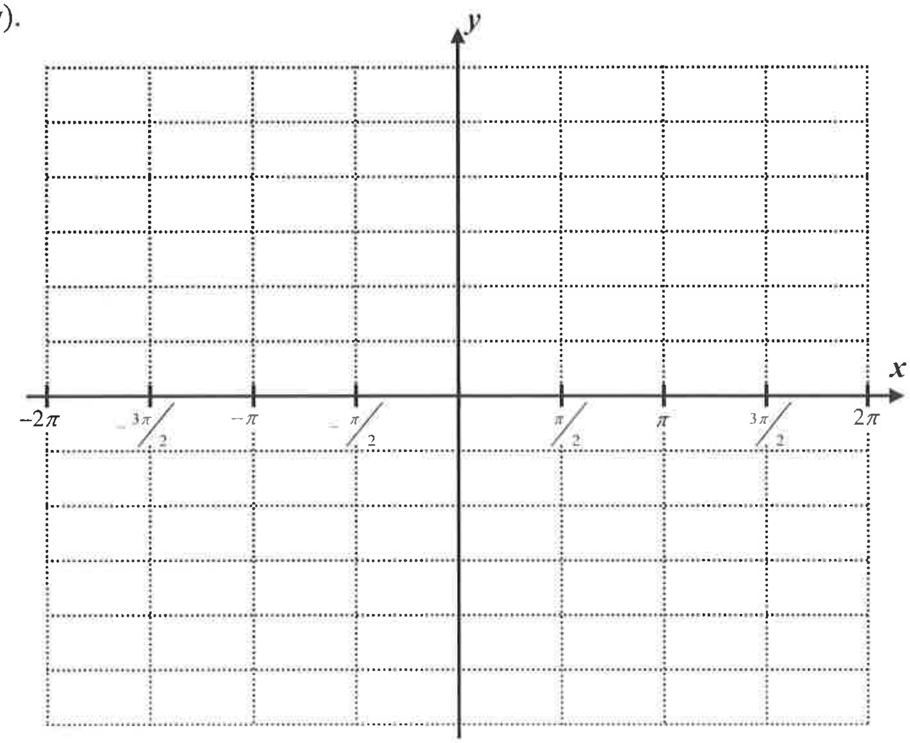


2. Graph and label both of the curves below. Then, state their intersection points (in other words, solve the system of equations shown below).

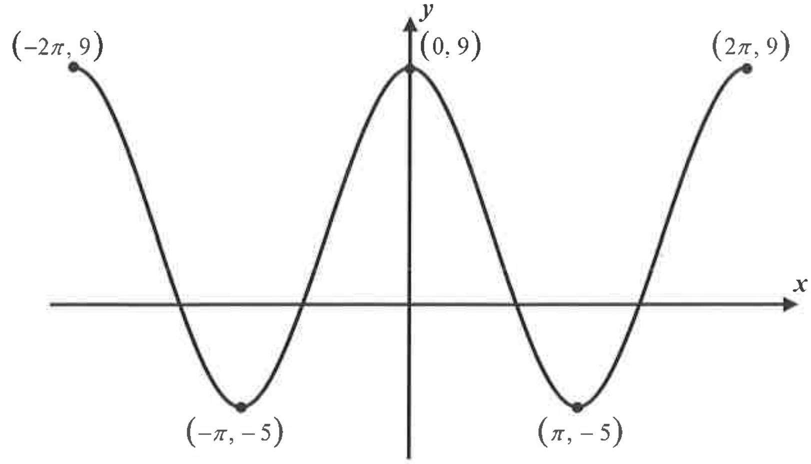
$y = 4\cos(x) + 1$

$y = -\cos(x) - 4$

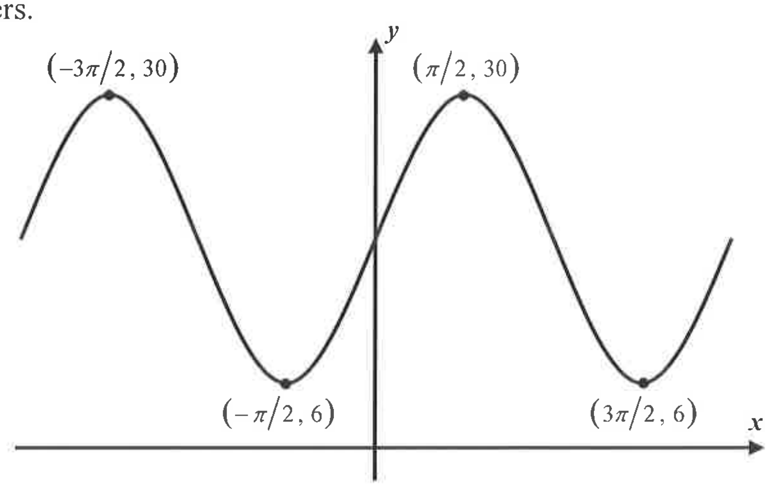
Intersection Points:



4. The following graph can be described using an equation of the form $y = A\cos(x) + C$. Determine the values of A and C. Show how you arrived at your answers.



5. The following graph can be described using an equation of the form $y = A\sin(x) + C$. Determine the values of A and C. Show how you arrived at your answers.



6. State the range of each of the following sinusoidal functions in interval form.

(a) $y = 10\sin(x) - 3$ (b) $y = -8\cos(x) + 2$ (c) $y = 22\sin(x) + 30$

7. When graphed, the line $y = 14$ would not intersect the graph of which of the following functions?

(1) $y = 5\cos(x) + 9$ (3) $y = 2\sin(x) + 15$

(2) $y = -6\cos(x) + 10$ (4) $y = 3\sin(x) + 20$

8. Which of the following functions has a maximum value of 25?

(1) $y = 25\sin(x) + 12$ (3) $y = 8\cos(x) + 17$

(2) $y = -10\cos(x) + 35$ (4) $y = 5\sin(x) + 15$

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11.8 Problem Set – The Frequency and Period of a Sinusoidal Graph

FLUENCY

1. For each of the following sinusoidal functions, determine its period in exact terms of pi.

(a) $y=6\sin(10x)$ (b) $y=-2\cos(8x)$ (c) $y=7\sin\left(\frac{1}{3}x\right)$

(d) $y=\frac{2}{3}\cos\left(\frac{4}{3}x\right)$ (e) $y=8\sin(0.25x)$ (f) $y=2.5\cos(0.4x)$

2. For each of the following sinusoidal functions, determine its exact period.

(a) $y=5\sin\left(\frac{2\pi}{7}x\right)$ (b) $y=5\cos\left(\frac{2\pi}{365}t\right)+12$ (c) $y=-8\sin\left(\frac{\pi}{9}x\right)-1$

3. If the period of a sinusoidal function is equal to 18, which of the following gives its frequency?

(1) $\frac{\pi}{9}$ (3) $\frac{\pi}{18}$

(2) 18π (4) 6π __________

4. It is known for that a particular sine curve repeats its fundamental pattern after every $\frac{2\pi}{7}$ units along the x-axis. Which of the following is the frequency of this curve?

(1) $\frac{2}{7}$ (3) $\frac{7}{2}$

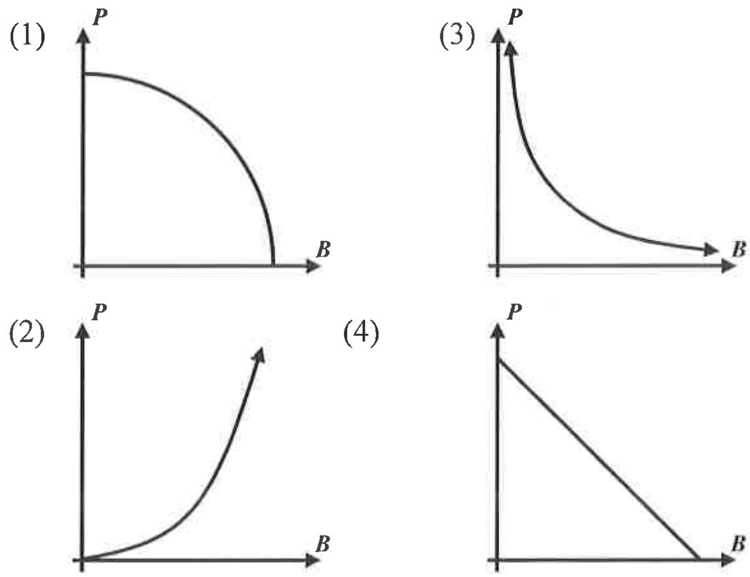
(2) 7 (4) 14 __________

5. When the period of a sine function doubles, the frequency

(1) doubles. (3) is halved.

(2) increases by 2. (4) decreases by 2.

6. Which of the following graphs shows the relationship between the frequency, B, and the period, P, of a sinusoidal graph? Experiment on your calculator. Graph the expression $P=\frac{2\pi}{B}$.



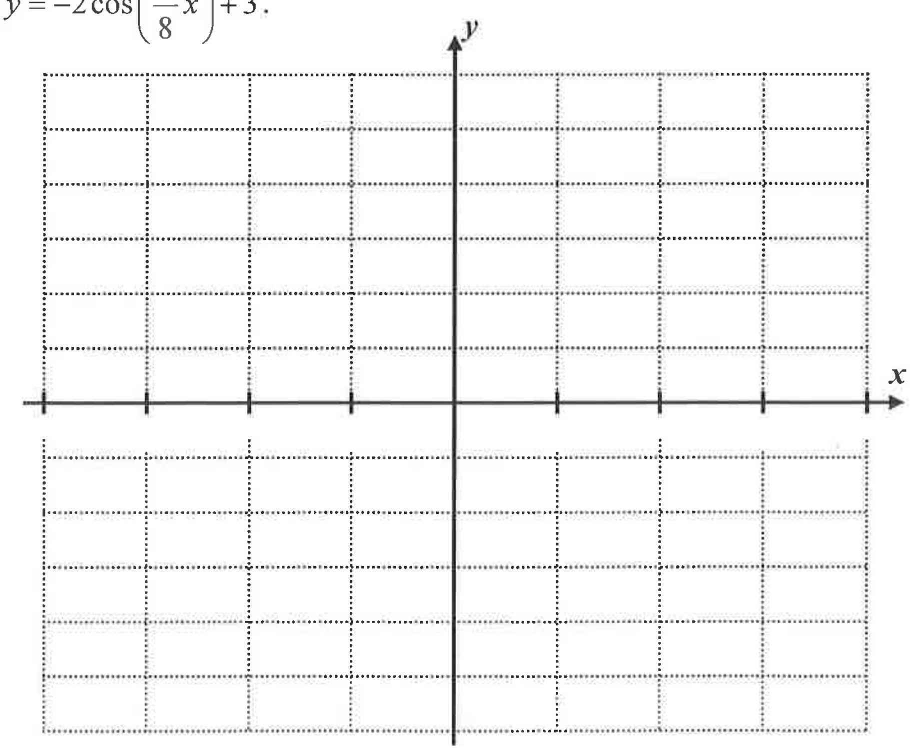
7. Consider the curve whose equation is $y=-2\cos\left(\frac{\pi}{8}x\right)+3$.

(a) Determine the exact period of this sinusoidal function.

(b) What is the amplitude of this sinusoidal function?

(c) What is the midline value of this sinusoidal function?

(d) Sketch the function on the axes for a full period on both sides of the y-axis. Label the scale on your x-axis.



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11.9 Problem Set – Sinusoidal Modeling

APPLICATIONS

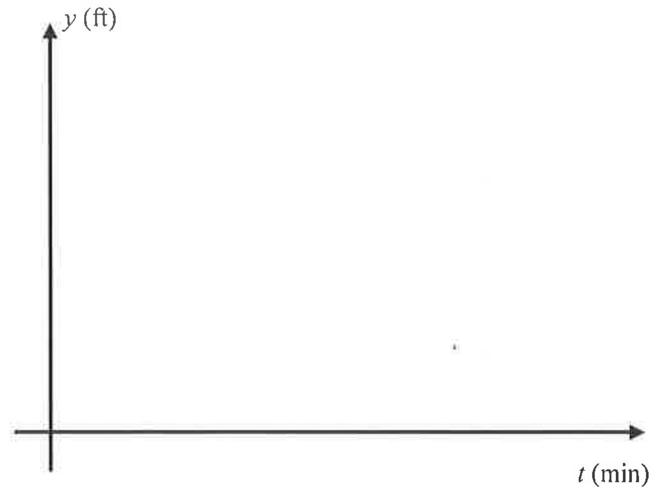
1. A ball is attached to a spring, which is stretched and then let go. The height of the ball is given by the sinusoidal equation $y = -3.5\cos\left(\frac{4\pi}{5}t\right) + 5$, where y is the height above the ground in feet and t is the number of seconds since the ball was released.

(a) At what height was the ball released at? Show the calculation that leads to your answer.

(b) What is the maximum height the ball reaches?

(c) How many seconds does it take the ball to return to its original position?

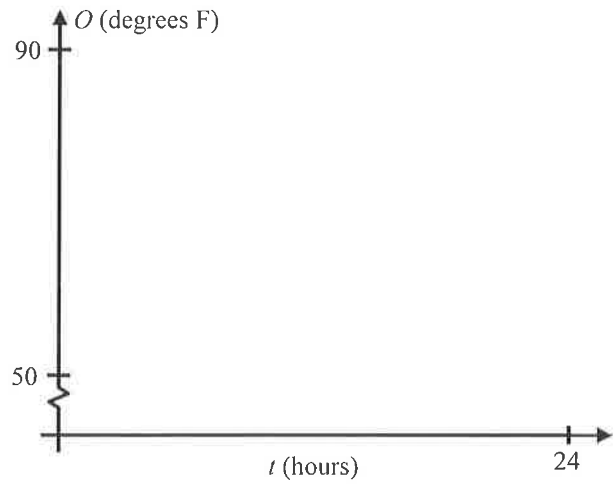
(d) Draw a rough sketch of one complete period of this curve below. Label maximum and minimum points.



2. An athlete was having her blood pressure monitored during a workout. Doctors found that her maximum blood pressure, known as systolic, was 110 and her minimum blood pressure, known as diastolic, was 70. If each heartbeat cycle takes 0.75 seconds, then determine a sinusoidal model, in the form $y = A\sin(Bt) + C$, for her blood pressure as a function of time t in seconds. Show the calculations that lead to your answer.

3. On a standard summer day in upstate New York, the temperature outside can be modeled using the sinusoidal equation $O(t)=11\cos\left(\frac{\pi}{12}t\right)+71$, where t represents the number of hours since the peak temperature for the day.

(a) Sketch a graph of this function on the axes below for one day.



(b) For $0\le t\le 24$, graphically determine all points in time when the outside temperature is equal to 75 degrees. Round your answers to the nearest tenth of an hour.

4. The percentage of the moon's surface that is visible to a person standing on the Earth varies with the time since the moon was full. The moon passes through a full cycle in 28 days, from full moon to full moon. The maximum percentage of the moon's surface that is visible is 50%. Determine an equation, in the form $P=A\cos(Bt)+C$ for the percentage of the surface that is visible, P, as a function of the number of days, t, since the moon was full. Show the work that leads to the values of A, B, and C.

5. Evie is on a swing thinking about trigonometry (no seriously!). She realizes that her height above the ground is a periodic function of time that can be modeled using $h=3\cos\left(\frac{\pi}{2}t\right)+5$, where t represents time in seconds. Which of the following is the range of Evie's heights?

(1) $2\le h\le 8$ (3) $3\le h\le 5$

(2) $4\le h\le 8$ (4) $2\le h\le 5$

______

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11.10 Problem Set – The Tangent Function

FLUENCY

1. Using the unit circle diagram, find the exact values for each of the following. Don't leave any complex fractions. Show how you arrived at your final answers. You can check using your calculator, but decimal answers should not be given. If the value of tangent is undefined, state UND.

(a) $\tan(60^\circ)$ (b) $\tan(150^\circ)$ (c) $\tan(225^\circ)$

(d) $\tan(270^\circ)$ (e) $\tan\left(\frac{2\pi}{3}\right)$ (f) $\tan\left(\frac{11\pi}{6}\right)$

2. The point $(0.28, 0.96)$ lies on the unit circle. Which of the following is closest to the tangent of an angle drawn in standard position whose terminal ray passes through this point?

(1) 3.43 (3) 0.29

(2) 1.73 (4) 0.42 ________

3. At which of the following angles is the tangent function undefined?

(1) $\theta = 180^\circ$ (3) $\theta = 45^\circ$

(2) $\theta = -90^\circ$ (4) $\theta = -360^\circ$ ________

4. Which of the following values of x is *not* in the domain of $g(x) = \tan(2x)$? Hint – you will be multiplying each of these values by 2 before finding its tangent.

(1) 45° (3) 180°

(2) 0° (4) 90° ________

5. Determine whether each function in the tables below is positive, $(+)$, or negative, $(-)$, for angles whose terminal rays lie in the respective quadrants. Use values of sine and cosine to determine the sign of the tangent function.

	I	II	III	IV
$\cos(\theta)$				
$\sin(\theta)$				
$\tan(\theta)$				

6. For an angle α it is known that $\tan(\alpha) > 0$ and $\sin(\alpha) < 0$. The terminal ray of α when drawn in standard position must lie in which quadrant? Hint: see the table in #5 for help.

(1) I (3) III

(2) II (4) IV

7. For each of the following problems, the value of either sine or cosine of an angle is given along with the quadrant in which the terminal ray of the angle lies. For each, produce the values of the two missing trigonometric functions. Some of your answers will have radicals (irrational numbers) in them. You should *not* leave complex fractions.

(a) $\cos(A) = \frac{-3}{5}$ and A terminates in quadrant II.

(b) $\sin(\alpha) = \frac{1}{3}$ and α terminates in quadrant I.

(c) $\sin(\theta) = -\frac{5}{13}$ and θ terminates in quadrant III.

(d) $\cos(B) = \frac{2}{5}$ and B terminates in quadrant IV.

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11.11 Problem Set – The Reciprocal Functions

FLUENCY

1. Determine the value of each of the following in exact and simplest form (leave no complex fractions).

(a) $\csc\left(30^\circ\right)$ (b) $\cot\left(90^\circ\right)$ (c) $\sec\left(180^\circ\right)$

(d) $\cot\left(\frac{\pi}{3}\right)$ (e) $\csc\left(\frac{3\pi}{2}\right)$ (f) $\sec\left(\frac{5\pi}{4}\right)$

2. Use your calculator to determine the value of each of the following to the nearest *hundredth*.

(a) $\cot\left(115^\circ\right)$ (b) $\sec\left(312^\circ\right)$ (c) $\csc\left(245^\circ\right)$

3. In simplest radical form, $\sec\left(135^\circ\right)$ is equal to

(1) $-\frac{\sqrt{2}}{3}$ (3) $-\frac{\sqrt{2}}{2}$

(2) $-\sqrt{2}$ (4) $-\frac{\sqrt{3}}{2}$ ________

4. Which of the following is nearest to the value of $\cot\left(220^\circ\right)$?

(1) 1.19 (3) −2.74

(2) 3.17 (4) −0.85 ________

5. For which of the following values of α is $\cot(\alpha)$ undefined? ________

(1) 60° (3) 180°

(2) 90° (4) 135°

6. For which angle, β, below will $\sec(\beta)$ not exist? ________

(1) 30° (3) 180°

(2) 45° (4) 90°

7. Determine whether each function in the tables below is positive, $(+)$, or negative, $(-)$, for angles whose terminal rays lie in the respective quadrants. Use the table in part (a) to help create the table in (b).

(a)

	I	II	III	IV
$\cos(\theta)$				
$\sin(\theta)$				

(b)

	I	II	III	IV
$\tan(\theta)$				
$\cot(\theta)$				
$\sec(\theta)$				
$\csc(\theta)$				

8. For the angle β it is known that $\csc(\beta)>0$ and $\sec(\beta)<0$. When drawn in standard position, the terminal ray of β lies in quadrant

(1) I (3) III

(2) II (4) IV

____

9. The angle θ when drawn in standard position has its terminal ray in the second quadrant. If it is known that $\sin\theta=\frac{5}{13}$ then determine the values of all of the remaining trigonometric functions.

(a) $\cos\theta$ (b) $\tan\theta$ (c) $\sec\theta$

(d) $\csc\theta$ (e) $\cot\theta$