

# ALGEBRA II (Common Core)

Wednesday, June 1, 2016 — 9:15 a.m. to 12:15 p.m., only

Student Name: \_\_\_\_\_

School Name: \_\_\_\_\_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

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## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for  
computations.

1 When  $b > 0$  and  $d$  is a positive integer, the expression  $(3b)^{\frac{2}{d}}$  is equivalent to

(1)  $\frac{1}{(\sqrt[d]{3b})^2}$

(3)  $\frac{1}{\sqrt{3b^d}}$

(2)  $(\sqrt{3b})^d$

(4)  $(\sqrt[d]{3b})^2$

2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests,  $T$ , are left in the semester?

(1)  $\frac{255 + 93T}{3T} = 90$

(3)  $\frac{255 + 93T}{T + 3} = 90$

(2)  $\frac{255 + 90T}{3T} = 93$

(4)  $\frac{255 + 90T}{T + 3} = 93$

3 Given  $i$  is the imaginary unit,  $(2 - yi)^2$  in simplest form is

(1)  $y^2 - 4yi + 4$

(3)  $-y^2 + 4$

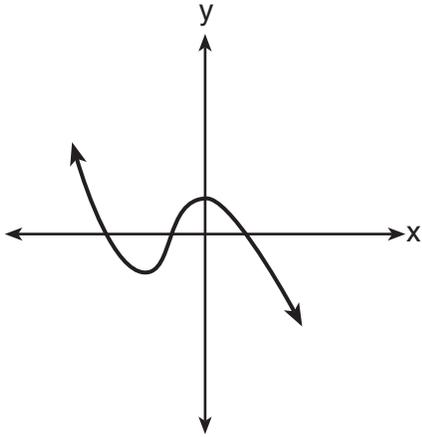
(2)  $-y^2 - 4yi + 4$

(4)  $y^2 + 4$

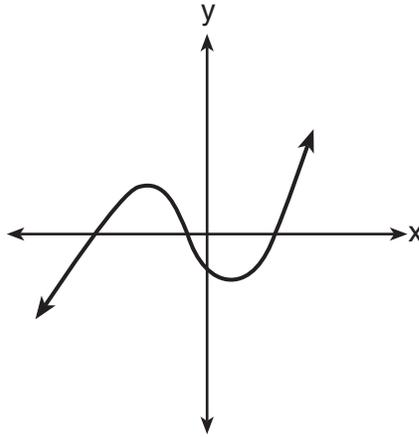
Use this space for computations.

4 Which graph has the following characteristics?

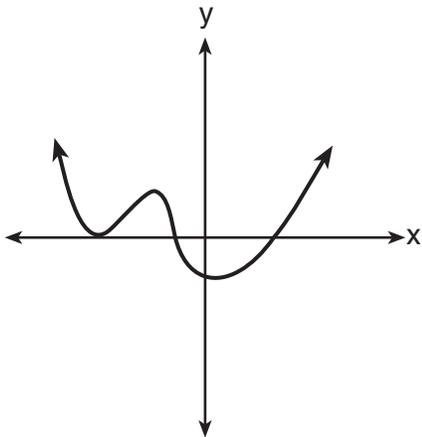
- three real zeros
- as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$
- as  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$



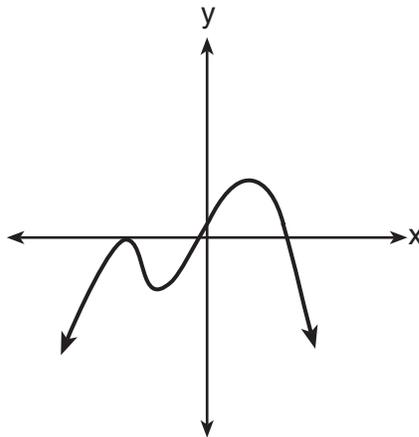
(1)



(3)



(2)



(4)

5 The solution set for the equation  $\sqrt{56 - x} = x$  is

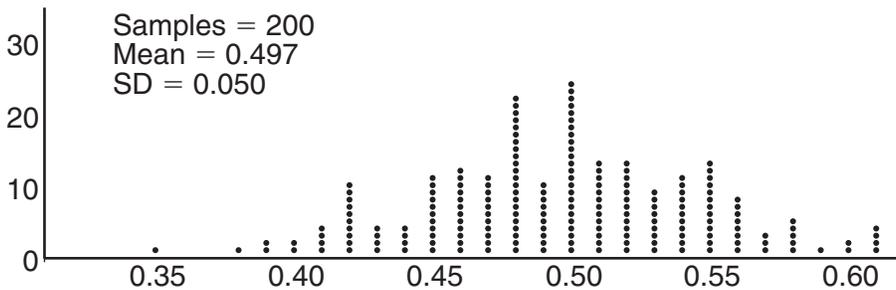
- |                 |             |
|-----------------|-------------|
| (1) $\{-8, 7\}$ | (3) $\{7\}$ |
| (2) $\{-7, 8\}$ | (4) $\{\}$  |

Use this space for computations.

6 The zeros for  $f(x) = x^4 - 4x^3 - 9x^2 + 36x$  are

- (1)  $\{0, \pm 3, 4\}$                       (3)  $\{0, \pm 3, -4\}$   
(2)  $\{0, 3, 4\}$                          (4)  $\{0, 3, -4\}$

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Given the results of her coin flips and of her computer simulation, which statement is most accurate?

- (1) 73 of the computer's next 100 coin flips will be heads.  
(2) 50 of her next 100 coin flips will be heads.  
(3) Her coin is not fair.  
(4) Her coin is fair.
- 8 If  $g(c) = 1 - c^2$  and  $m(c) = c + 1$ , then which statement is *not* true?

- (1)  $g(c) \cdot m(c) = 1 + c - c^2 - c^3$   
(2)  $g(c) + m(c) = 2 + c - c^2$   
(3)  $m(c) - g(c) = c + c^2$   
(4)  $\frac{m(c)}{g(c)} = \frac{-1}{1 - c}$



**Use this space for  
computations.**

**12** A solution of the equation  $2x^2 + 3x + 2 = 0$  is

(1)  $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$                       (3)  $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

(2)  $-\frac{3}{4} + \frac{7}{4}i$                       (4)  $\frac{1}{2}$

**13** The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height,  $H$ , in feet, above the ground of one of the six-person cars can be modeled by

$H(t) = 70 \sin\left(\frac{2\pi}{7}(t - 1.75)\right) + 80$ , where  $t$  is time, in minutes. Using

$H(t)$  for one full rotation, this car's minimum height, in feet, is

(1) 150                                      (3) 10

(2) 70                                        (4) 0

**14** The expression  $\frac{4x^3 + 5x + 10}{2x + 3}$  is equivalent to

(1)  $2x^2 + 3x - 7 + \frac{31}{2x + 3}$                       (3)  $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$

(2)  $2x^2 - 3x + 7 - \frac{11}{2x + 3}$                       (4)  $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$

**15** Which function represents exponential decay?

(1)  $y = 2^{0.3t}$                                       (3)  $y = \left(\frac{1}{2}\right)^{-t}$

(2)  $y = 1.2^{3t}$                                       (4)  $y = 5^{-t}$

Use this space for  
computations.

16 Given  $f^{-1}(x) = -\frac{3}{4}x + 2$ , which equation represents  $f(x)$ ?

- (1)  $f(x) = \frac{4}{3}x - \frac{8}{3}$                       (3)  $f(x) = \frac{3}{4}x - 2$   
(2)  $f(x) = -\frac{4}{3}x + \frac{8}{3}$                       (4)  $f(x) = -\frac{3}{4}x + 2$

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle,  $\theta$ , intercepts the circle in Quadrant II at point  $C$ . The  $y$ -coordinate of point  $C$  is 8. What is the value of  $\cos \theta$ ?

- (1)  $-\frac{3}{5}$     (3)  $\frac{3}{5}$   
(2)  $-\frac{3}{4}$     (4)  $\frac{4}{5}$

18 Which statement about the graph of  $c(x) = \log_6 x$  is *false*?

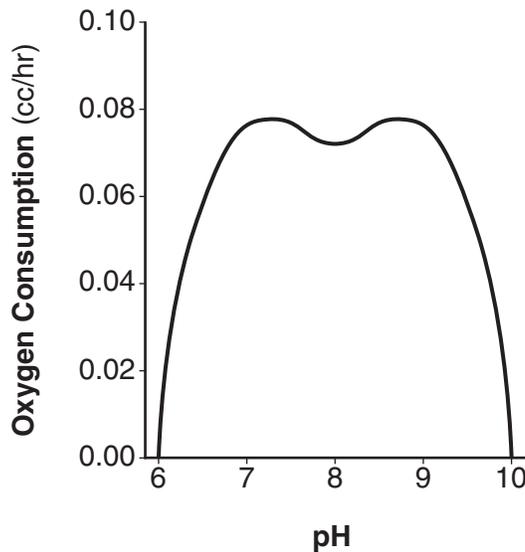
- (1) The asymptote has equation  $y = 0$ .  
(2) The graph has no  $y$ -intercept.  
(3) The domain is the set of positive reals.  
(4) The range is the set of all real numbers.

19 The equation  $4x^2 - 24x + 4y^2 + 72y = 76$  is equivalent to

- (1)  $4(x - 3)^2 + 4(y + 9)^2 = 76$   
(2)  $4(x - 3)^2 + 4(y + 9)^2 = 121$   
(3)  $4(x - 3)^2 + 4(y + 9)^2 = 166$   
(4)  $4(x - 3)^2 + 4(y + 9)^2 = 436$

Use this space for  
computations.

- 20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.



Which statement about this function is *incorrect*?

- (1) The degree of the polynomial is even.
  - (2) There is a positive leading coefficient.
  - (3) At two pH values, there is a relative maximum value.
  - (4) There are two intervals where the function is decreasing.
- 21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let  $m$  represent months.]

- (1)  $(1.0525)^m$
- (2)  $(1.0525)^{\frac{12}{m}}$
- (3)  $(1.00427)^m$
- (4)  $(1.00427)^{\frac{m}{12}}$



## Part II

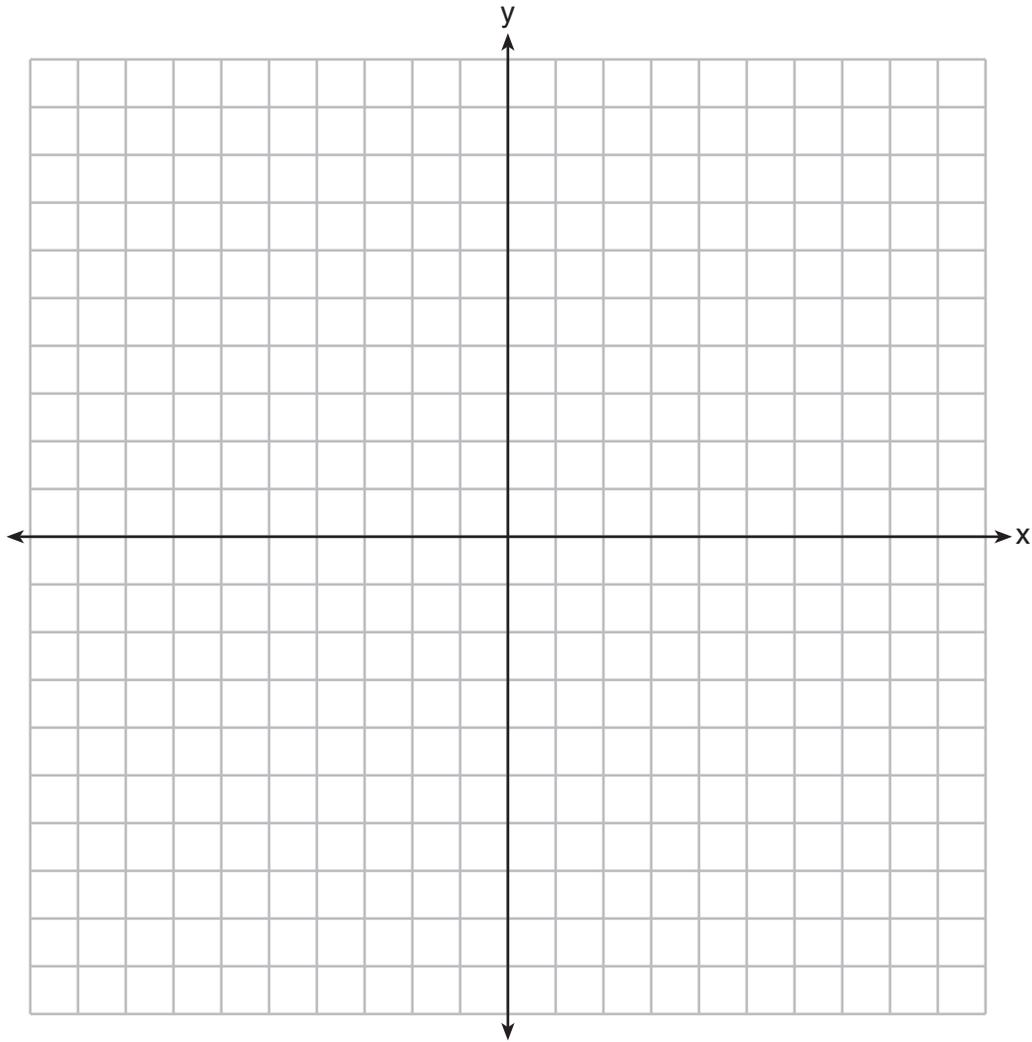
Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Solve for  $x$ :  $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

**26** Describe how a controlled experiment can be created to examine the effect of ingredient  $X$  in a toothpaste.

**27** Determine if  $x - 5$  is a factor of  $2x^3 - 4x^2 - 7x - 10$ . Explain your answer.

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period  $\frac{\pi}{2}$ ,  
midline  $y = -1$ , and passing through the point  $(0,2)$ .



**29** A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is  $\frac{974}{1376}$ , what is the probability that a student participates in both sports and music?

**30** The directrix of the parabola  $12(y + 3) = (x - 4)^2$  has the equation  $y = -6$ . Find the coordinates of the focus of the parabola.

**31** Algebraically prove that  $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$ , where  $x \neq -2$ .

**32** A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

### Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$

$$2x + 2y = 10$$

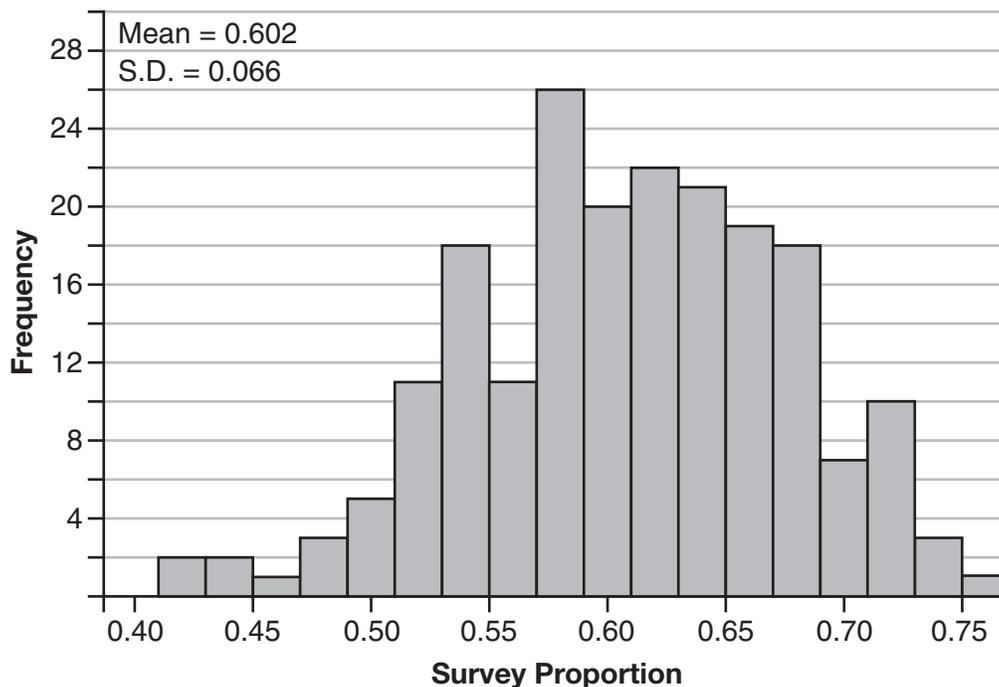
**34** Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula,  $S_n$ , for Alexa's total earnings over  $n$  years.

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

**35** Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

36 Which function shown below has a greater average rate of change on the interval  $[-2, 4]$ ? Justify your answer.

$x$	$f(x)$
-4	0.3125
-3	0.625
-2	1.25
-1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$$g(x) = 4x^3 - 5x^2 + 3$$

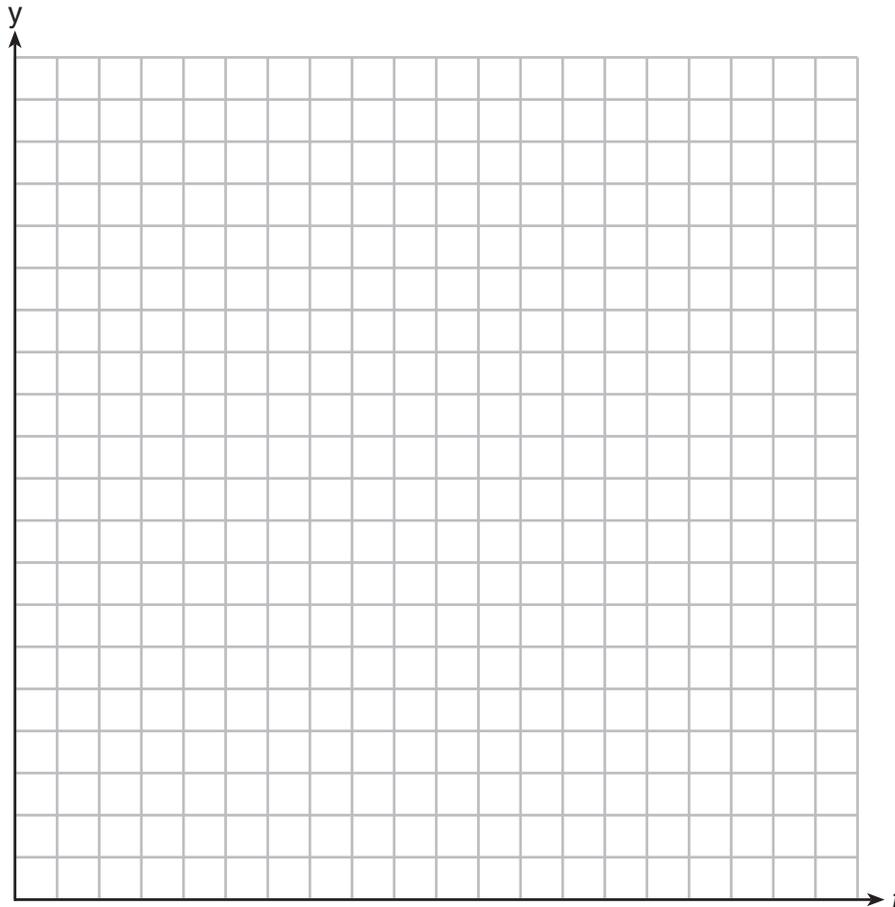
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function  $N(t) = N_0(e)^{-rt}$ , where  $N(t)$  is the amount left in the body,  $N_0$  is the initial dosage,  $r$  is the decay rate, and  $t$  is time in hours. Patient A,  $A(t)$ , is given 800 milligrams of a drug with a decay rate of 0.347. Patient B,  $B(t)$ , is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions,  $A(t)$  and  $B(t)$ , to represent the breakdown of the respective drug given to each patient.

Graph each function on the set of axes below.

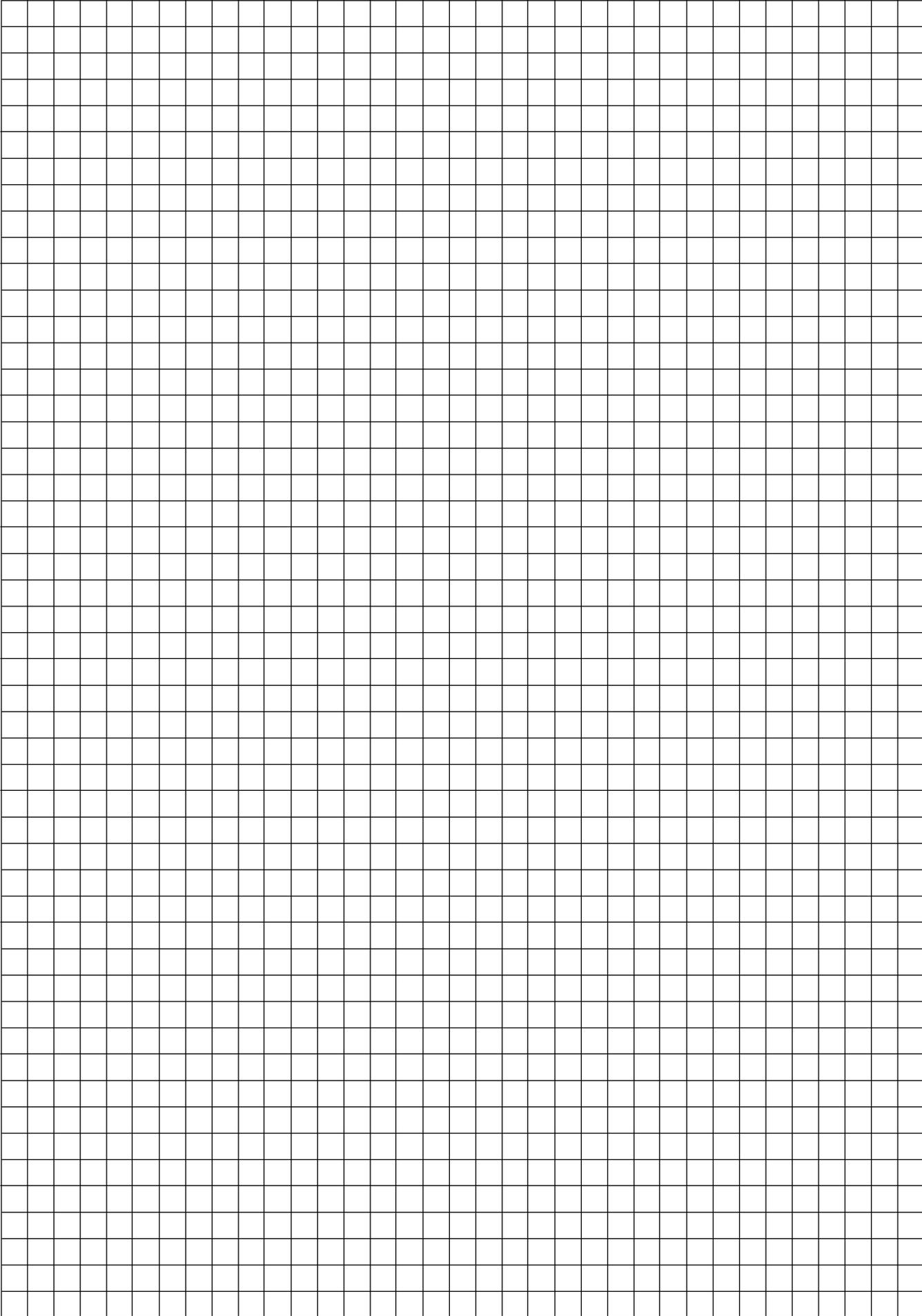


To the *nearest hour*,  $t$ , when does the amount of the given drug remaining in patient  $B$  begin to exceed the amount of the given drug remaining in patient  $A$ ?

The doctor will allow patient  $A$  to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient  $A$  will have to wait to take another 800 milligram dose of the drug.



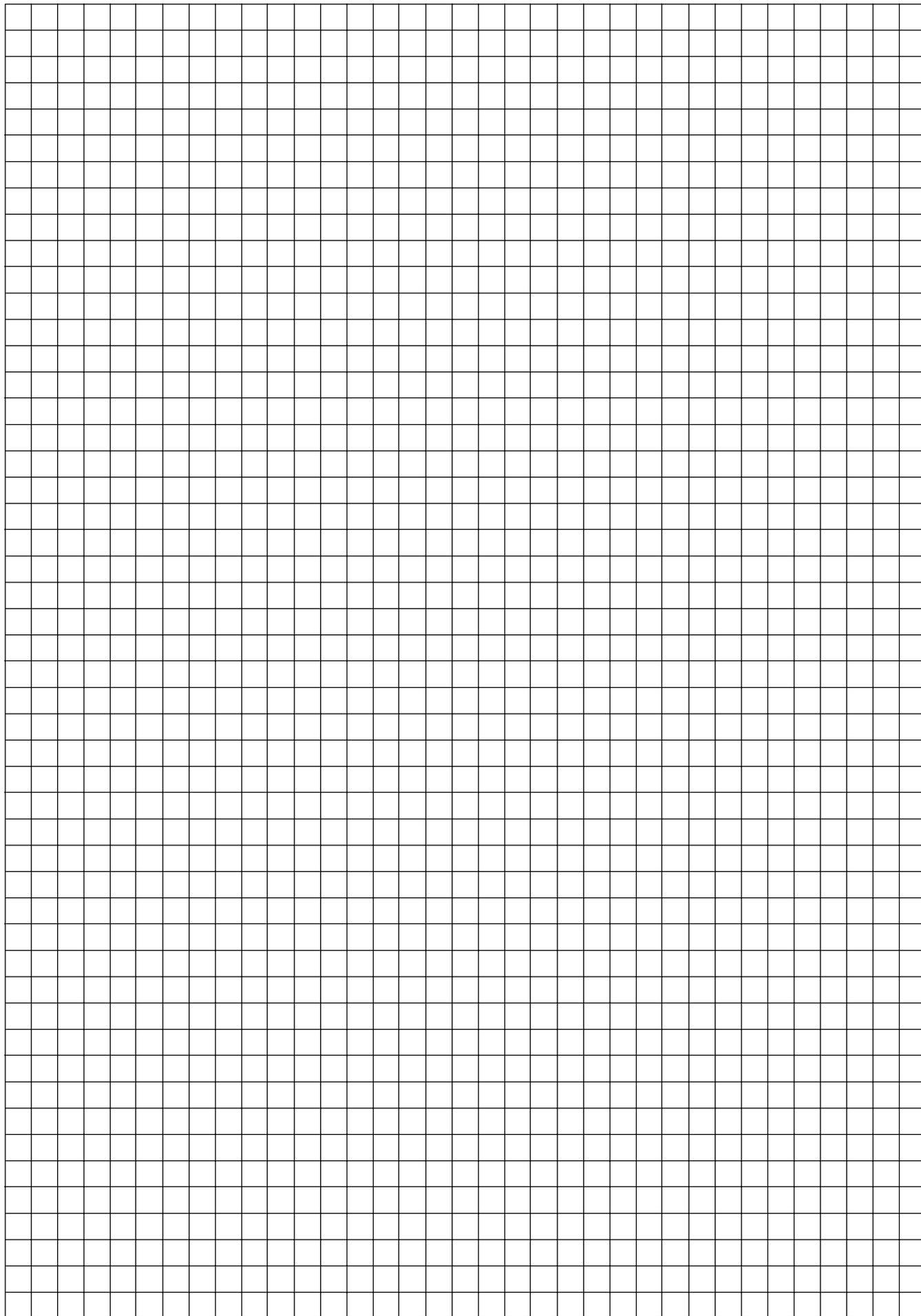
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Scrap Graph Paper — This sheet will *not* be scored.



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## High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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# ALGEBRA II (Common Core)

Thursday, August 18, 2016 — 12:30 to 3:30 p.m., only

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School Name: \_\_\_\_\_

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## Part I

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Use this space for  
computations.

1 Which equation has  $1 - i$  as a solution?

(1)  $x^2 + 2x - 2 = 0$

(3)  $x^2 - 2x - 2 = 0$

(2)  $x^2 + 2x + 2 = 0$

(4)  $x^2 - 2x + 2 = 0$

2 Which statement(s) about statistical studies is true?

I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.

II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.

III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.

IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math.

(1) I, only

(3) I and III

(2) II, only

(4) III and IV



**Use this space for computations.**

6 Sally's high school is planning their spring musical. The revenue,  $R$ , generated can be determined by the function  $R(t) = -33t^2 + 360t$ , where  $t$  represents the price of a ticket. The production cost,  $C$ , of the musical is represented by the function  $C(t) = 700 + 5t$ . What is the highest ticket price, to *the nearest dollar*, they can charge in order to *not* lose money on the event?

- (1)  $t = 3$                                       (3)  $t = 8$   
(2)  $t = 5$                                       (4)  $t = 11$

7 The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

Age Group	Text Messages per Month		
	0–10	11–50	Over 50
15–18	4	37	68
19–22	6	25	87
23–60	25	47	157

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

- (1)  $\frac{157}{229}$                                       (3)  $\frac{157}{384}$   
(2)  $\frac{157}{312}$                                       (4)  $\frac{157}{456}$

8 A recursive formula for the sequence 18, 9, 4.5, ... is

- (1)  $g_1 = 18$                                       (3)  $g_1 = 18$   
 $g_n = \frac{1}{2}g_{n-1}$                                        $g_n = 2g_{n-1}$   
(2)  $g_n = 18\left(\frac{1}{2}\right)^{n-1}$                                       (4)  $g_n = 18(2)^{n-1}$

**Use this space for  
computations.**

**9** Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

(1)  $\sum_{n=1}^6 8(1.10)^{n-1}$

(3)  $\frac{8 - 8(1.10)^6}{0.90}$

(2)  $\sum_{n=1}^6 8(1.10)^n$

(4)  $\frac{8 - 8(0.10)^n}{1.10}$

**10** A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave *decreasing*, only?

(1) (0, 200)

(3) (200, 400)

(2) (100, 300)

(4) (300, 400)

**11** The expression  $\frac{x^3 + 2x^2 + x + 6}{x + 2}$  is equivalent to

(1)  $x^2 + 3$

(3)  $2x^2 + x + 6$

(2)  $x^2 + 1 + \frac{4}{x + 2}$

(4)  $2x^2 + 1 + \frac{4}{x + 2}$



Use this space for computations.

14 Which equation represents an odd function?

(1)  $y = \sin x$

(3)  $y = (x + 1)^3$

(2)  $y = \cos x$

(4)  $y = e^{5x}$

15 The completely factored form of  $2d^4 + 6d^3 - 18d^2 - 54d$  is

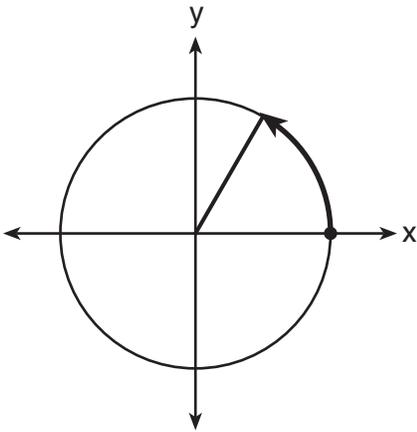
(1)  $2d(d^2 - 9)(d + 3)$

(3)  $2d(d + 3)^2(d - 3)$

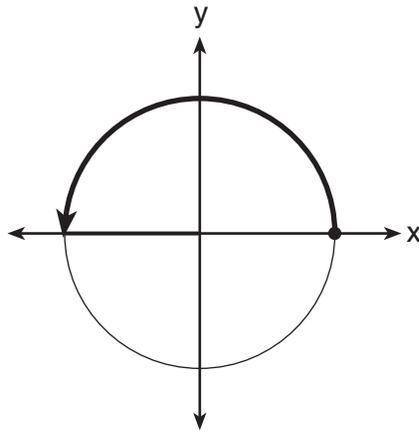
(2)  $2d(d^2 + 9)(d + 3)$

(4)  $2d(d - 3)^2(d + 3)$

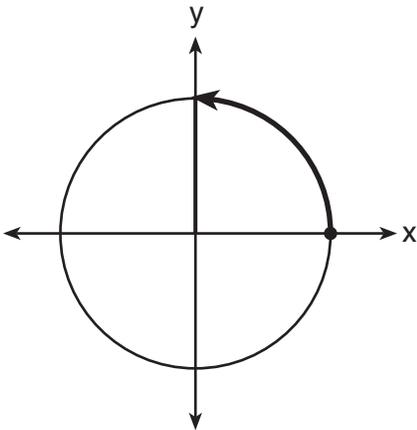
16 Which diagram shows an angle rotation of 1 radian on the unit circle?



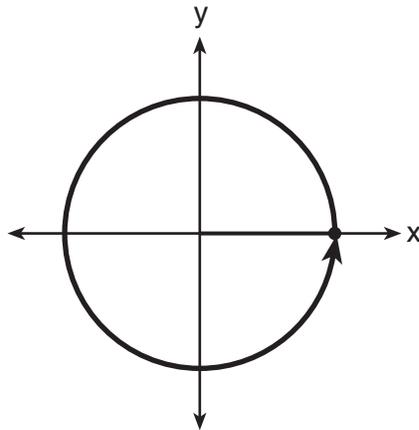
(1)



(3)



(2)



(4)



Use this space for  
computations.

20 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I  $(m + p)^2 = m^2 + 2mp + p^2$

II  $(x + y)^3 = x^3 + 3xy + y^3$

III  $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

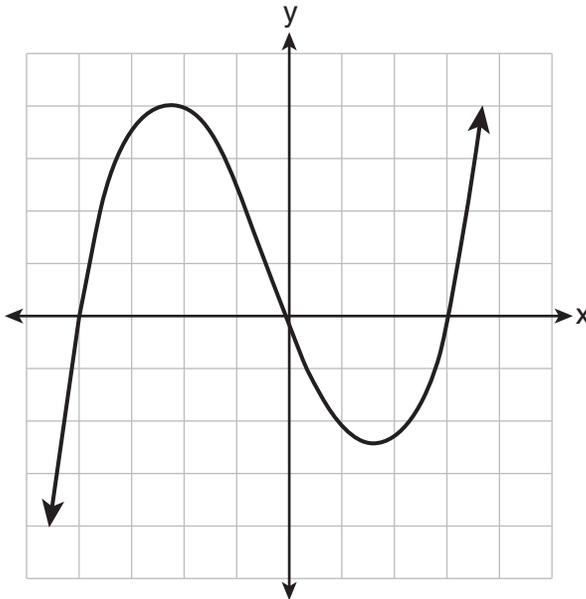
(1) I, only

(3) II and III

(2) I and II

(4) I and III

21 The graph of  $p(x)$  is shown below.



What is the remainder when  $p(x)$  is divided by  $x + 4$ ?

(1)  $x - 4$

(3) 0

(2)  $-4$

(4) 4

**Use this space for computations.**

**22** A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

(1)  $300(.30)^{\frac{14}{365}}$

(3)  $300(.30)^{\frac{365}{14}}$

(2)  $300(1.30)^{\frac{14}{365}}$

(4)  $300(1.30)^{\frac{365}{14}}$

**23** Which value is *not* contained in the solution of the system shown below?

$$a + 5b - c = -20$$

$$4a - 5b + 4c = 19$$

$$-a - 5b - 5c = 2$$

(1)  $-2$

(3)  $3$

(2)  $2$

(4)  $-3$

**24** In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State  $t$  years after 2010?

(1)  $P_t = 19,378,000(1.5)^t$

(2)  $P_0 = 19,378,000$

$$P_t = 19,378,000 + 1.015P_{t-1}$$

(3)  $P_t = 19,378,000(1.015)^{t-1}$

(4)  $P_0 = 19,378,000$

$$P_t = 1.015P_{t-1}$$

---

## Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 25 The volume of air in a person's lungs, as the person breathes in and out, can be modeled by a sine graph. A scientist is studying the differences in this volume for people at rest compared to people told to take a deep breath. When examining the graphs, should the scientist focus on the amplitude, period, or midline? Explain your choice.

**26** Explain how  $\left(3^{\frac{1}{5}}\right)^2$  can be written as the equivalent radical expression  $\sqrt[5]{9}$ .

27 Simplify  $xi(i - 7i)^2$ , where  $i$  is the imaginary unit.

**28** Using the identity  $\sin^2 \theta + \cos^2 \theta = 1$ , find the value of  $\tan \theta$ , to the *nearest hundredth*, if  $\cos \theta$  is  $-0.7$  and  $\theta$  is in Quadrant II.

**29** Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again.

A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth's wait time unusual? Justify your answer.

**30** The  $x$ -value of which function's  $x$ -intercept is larger,  $f$  or  $h$ ? Justify your answer.

$$f(x) = \log(x - 4)$$

$x$	$h(x)$
-1	6
0	4
1	2
2	0
3	-2

**31** The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.

<b>Speed</b> (mph)	10	20	30	40	50	60	70
<b>Distance</b> (ft)	6.25	25	56.25	100	156.25	225	306.25

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph.

Explain what this rate of change means as it relates to braking distance.

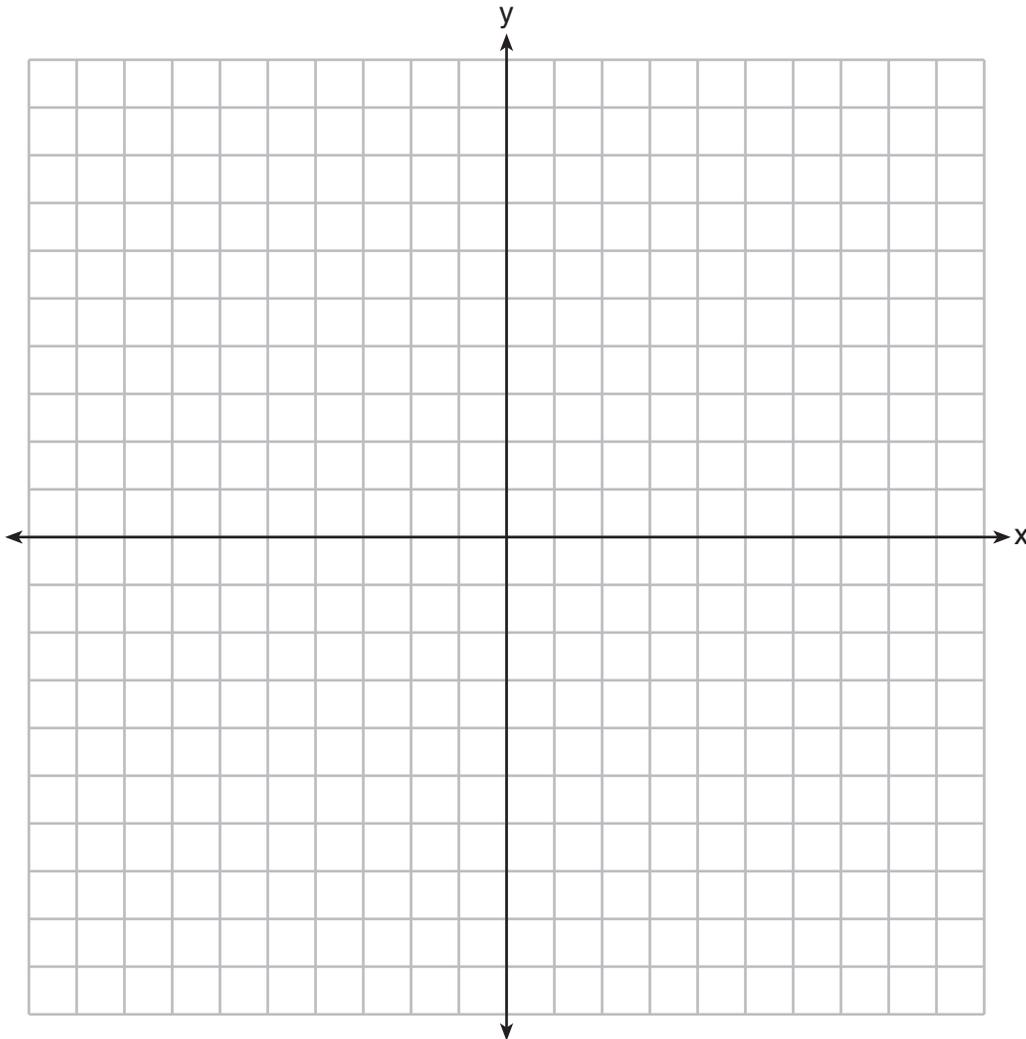
**32** Given events  $A$  and  $B$ , such that  $P(A) = 0.6$ ,  $P(B) = 0.5$ , and  $P(A \cup B) = 0.8$ , determine whether  $A$  and  $B$  are independent or dependent.

**Part III**

**Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]**

**33** Find algebraically the zeros for  $p(x) = x^3 + x^2 - 4x - 4$ .

On the set of axes below, graph  $y = p(x)$ .



**34** One of the medical uses of Iodine-131 ( $I-131$ ), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of  $I-131$  is approximately 8.02 days. A patient is injected with 20 milligrams of  $I-131$ . Determine, to the *nearest day*, the amount of time needed before the amount of  $I-131$  in the patient's body is approximately 7 milligrams.

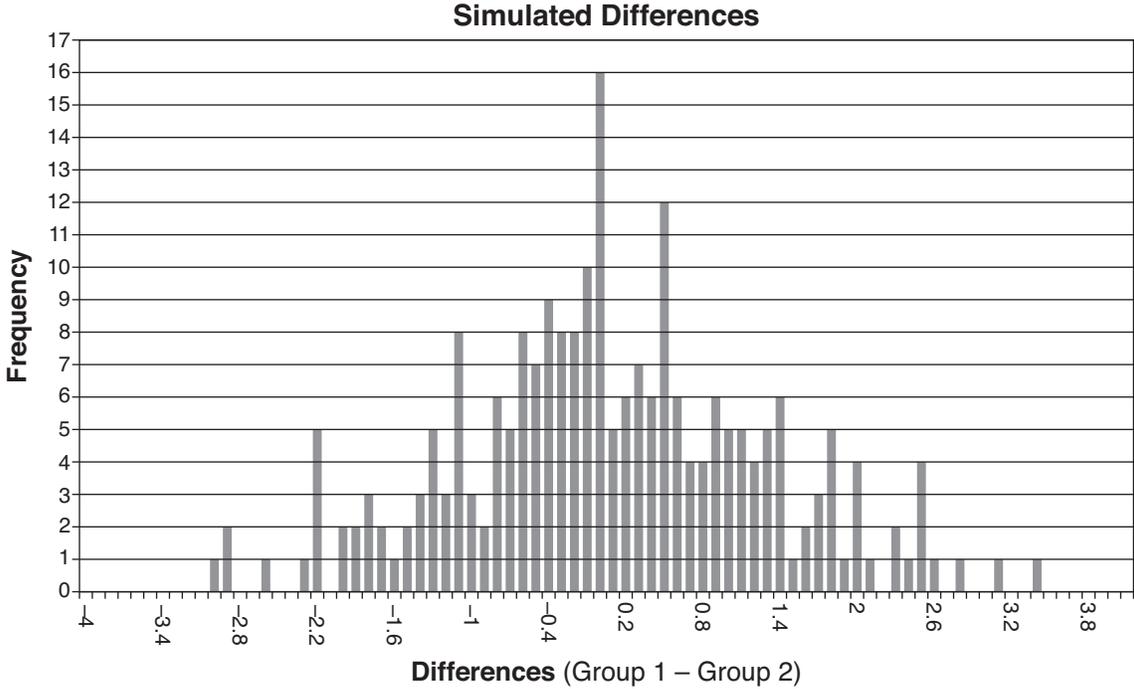
**35** Solve the equation  $\sqrt{2x-7} + x = 5$  algebraically, and justify the solution set.

**36** Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

<b>Group 1</b> (seconds)	<b>Group 2</b> (seconds)
17.4	23.3
18.1	18.8
18.2	22.1
19.6	12.7
18.6	16.9
16.2	24.4
16.1	21.2
15.3	21.2
17.8	16.3
19.7	14.5
Mean = 17.7	Mean = 19.1

- a) Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva's hypothesis may be *incorrect*.

Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.



b) Ayva has decided that the difference in mean reading times is *not* an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.

## Part IV

**Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]**

**37** Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option *A* will pay him 4.5% interest compounded annually. Option *B* will pay him 4.6% compounded quarterly.

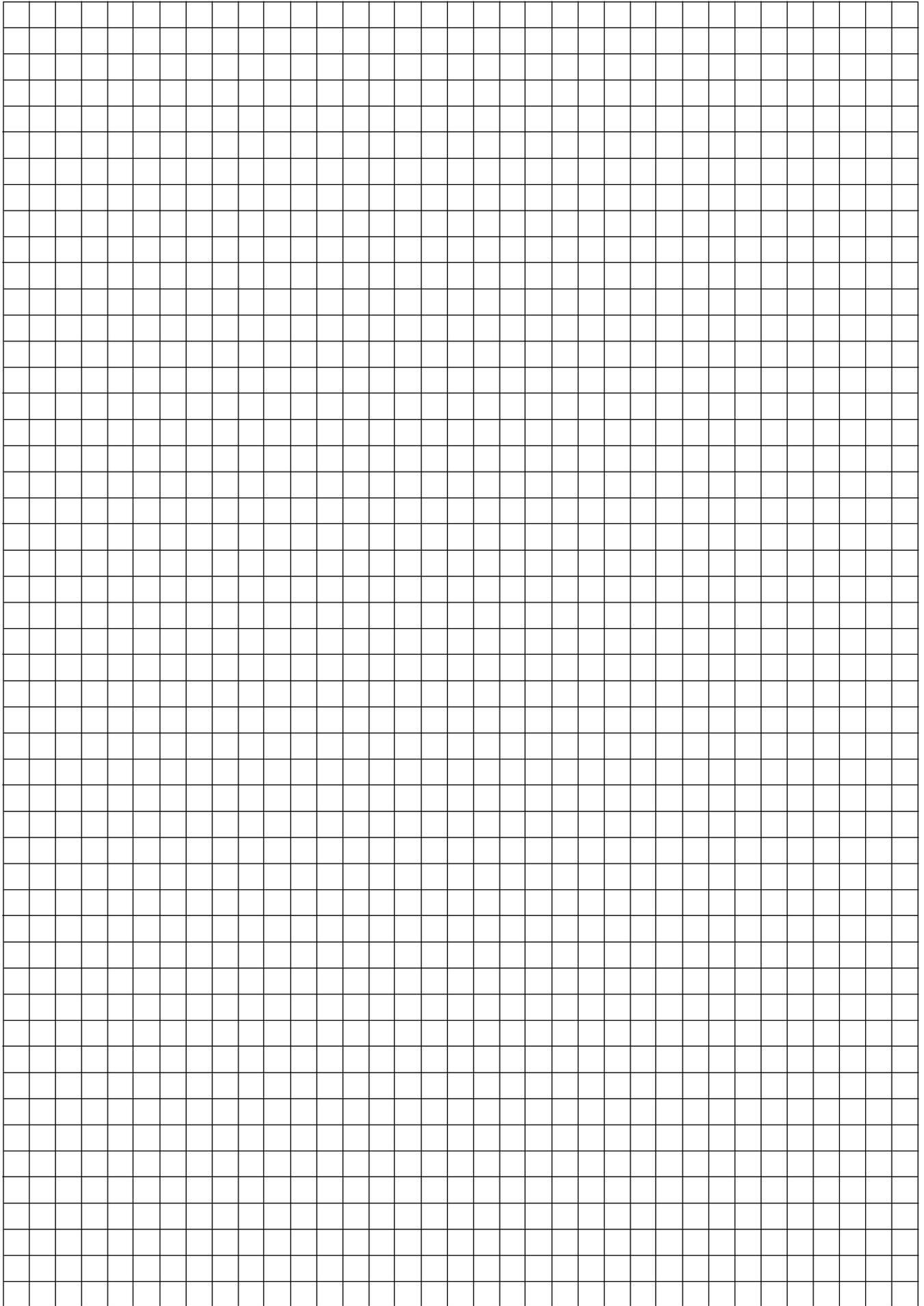
Write a function of option *A* and option *B* that calculates the value of each account after  $n$  years.

Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option *B* will earn than option *A* to the *nearest cent*.

Algebraically determine, to the *nearest tenth of a year*, how long it would take for option *B* to double Seth's initial investment.

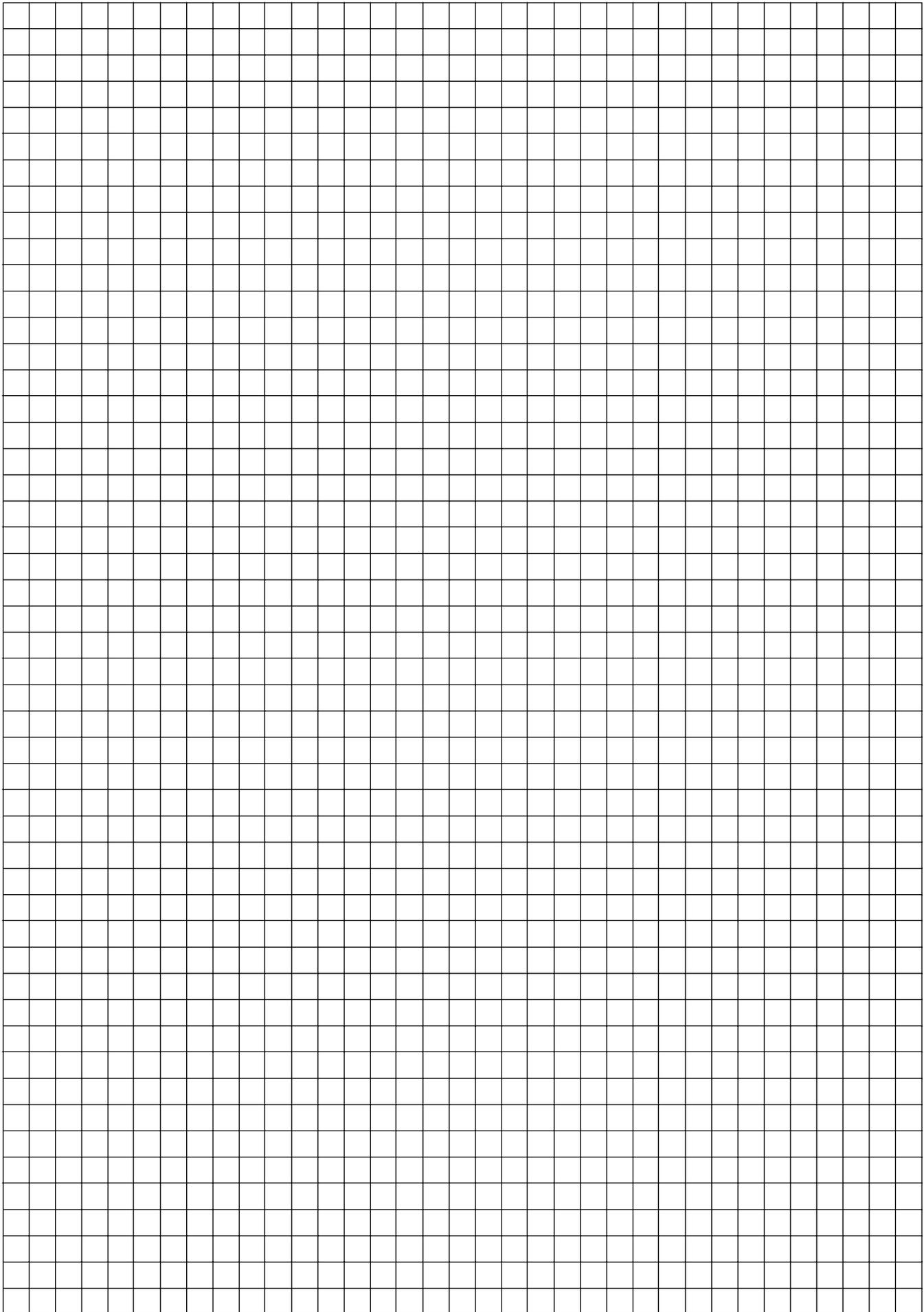
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## High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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# ALGEBRA II (Common Core)

Friday, January 27, 2017 — 9:15 a.m. to 12:15 p.m., only.

Student Name: \_\_\_\_\_

School Name: \_\_\_\_\_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for  
computations.

1 Relative to the graph of  $y = 3\sin x$ , what is the shift of the graph of

$$y = 3\sin\left(x + \frac{\pi}{3}\right)?$$

- (1)  $\frac{\pi}{3}$  right                      (3)  $\frac{\pi}{3}$  up  
(2)  $\frac{\pi}{3}$  left                        (4)  $\frac{\pi}{3}$  down

2 A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If  $t$  represents the time, in weeks, and  $P(t)$  is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?

- (1) 56                                  (3) 3688  
(2) 152                                (4) 81,920

3 When factored completely,  $m^5 + m^3 - 6m$  is equivalent to

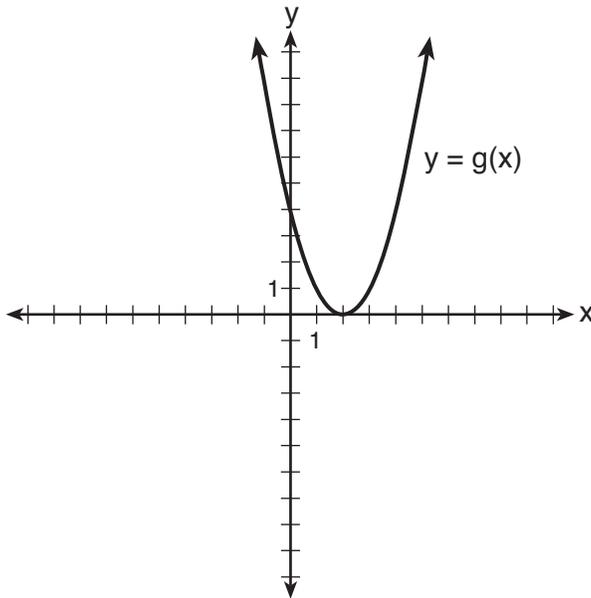
- (1)  $(m + 3)(m - 2)$               (3)  $m(m^4 + m^2 - 6)$   
(2)  $(m^3 + 3m)(m^2 - 2)$         (4)  $m(m^2 + 3)(m^2 - 2)$

4 If  $\sin^2(32^\circ) + \cos^2(M) = 1$ , then  $M$  equals

- (1)  $32^\circ$                                 (3)  $68^\circ$   
(2)  $58^\circ$                                 (4)  $72^\circ$

**Use this space for computations.**

- 5 What is the solution to the system of equations  $y = 3x - 2$  and  $y = g(x)$  where  $g(x)$  is defined by the function below?



- (1)  $\{(0, -2)\}$  (3)  $\{(1, 6)\}$   
(2)  $\{(0, -2), (1, 6)\}$  (4)  $\{(1, 1), (6, 16)\}$

- 6 Which statement about statistical analysis is *false*?

- (1) Experiments can suggest patterns and relationships in data.  
(2) Experiments can determine cause and effect relationships.  
(3) Observational studies can determine cause and effect relationships.  
(4) Observational studies can suggest patterns and relationships in data.

- 7 The expression  $\left(\frac{m^2}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$  is equivalent to

- (1)  $-\sqrt[6]{m^5}$  (3)  $-m^{\frac{5}{6}}$   
(2)  $\frac{1}{\sqrt[6]{m^5}}$  (4)  $\frac{1}{m^{\frac{5}{6}}}$



Use this space for computations.

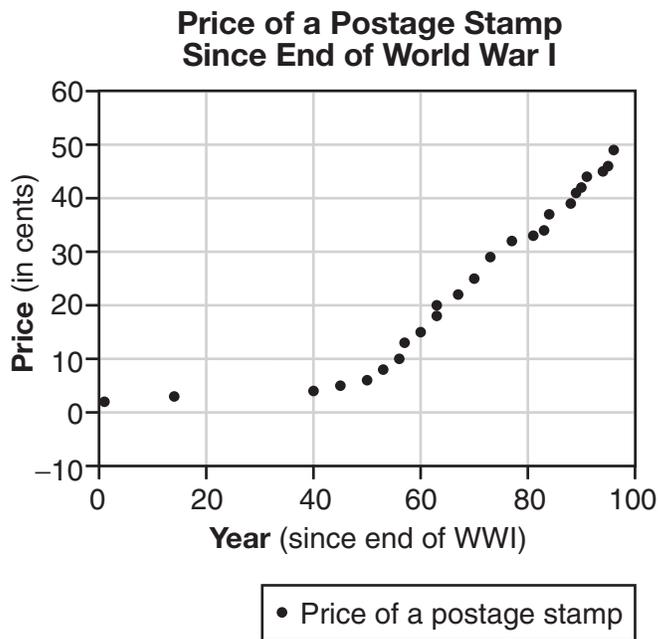
11 The solution to the equation  $18x^2 - 24x + 87 = 0$  is

- (1)  $-\frac{2}{3} \pm 6i\sqrt{158}$                       (3)  $\frac{2}{3} \pm 6i\sqrt{158}$   
(2)  $-\frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$                       (4)  $\frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$

12 When  $g(x) = \frac{2}{x+2}$  and  $h(x) = \log(x + 1) + 3$  are graphed on the same set of axes, which coordinates best approximate their point of intersection?

- (1)  $(-0.9, 1.8)$                               (3)  $(1.4, 3.3)$   
(2)  $(-0.9, 1.9)$                               (4)  $(1.4, 3.4)$

13 The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.



The equation that best models the price, in cents, of a postage stamp based on these data is

- (1)  $y = 0.59x - 14.82$                       (3)  $y = 1.43(1.04)^x$   
(2)  $y = 1.04(1.43)^x$                       (4)  $y = 24\sin(14x) + 25$



Use this space for computations.

17 What is the solution, if any, of the equation

$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12} ?$$

- (1)  $-1$  (3) all real numbers  
(2)  $-5$  (4) no real solution

18 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores?

- (1)  $430 \pm 115$  (3)  $496 \pm 115$   
(2)  $430 \pm 230$  (4)  $496 \pm 230$

19 Which statement regarding the graphs of the functions below is *untrue*?

$$\begin{aligned} f(x) &= 3 \sin 2x, \text{ from } -\pi < x < \pi & h(x) &= \log_2 x \\ g(x) &= (x - 0.5)(x + 4)(x - 2) & j(x) &= -|4x - 2| + 3 \end{aligned}$$

- (1)  $f(x)$  and  $j(x)$  have a maximum  $y$ -value of 3.  
(2)  $f(x)$ ,  $h(x)$ , and  $j(x)$  have one  $y$ -intercept.  
(3)  $g(x)$  and  $j(x)$  have the same end behavior as  $x \rightarrow -\infty$ .  
(4)  $g(x)$ ,  $h(x)$ , and  $j(x)$  have rational zeros.

20 When  $g(x)$  is divided by  $x + 4$ , the remainder is 0. Given  $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$ , which conclusion about  $g(x)$  is true?

- (1)  $g(4) = 0$   
(2)  $g(-4) = 0$   
(3)  $x - 4$  is a factor of  $g(x)$ .  
(4) No conclusion can be made regarding  $g(x)$ .

**Use this space for computations.**

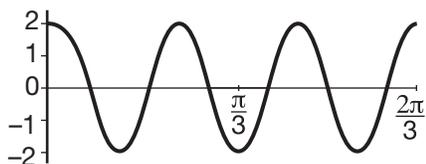
- 21 Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of  $B$  dollars after  $m$  months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after  $m$  months.

$m$	$B$
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

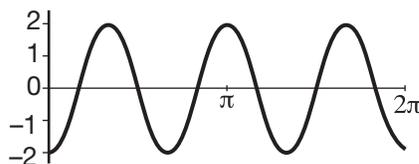
Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

- (1) month 10 to month 60      (3) month 36 to month 72  
 (2) month 19 to month 69      (4) month 60 to month 73

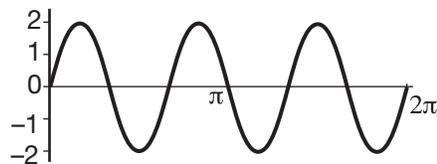
- 22 Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of  $\frac{2\pi}{3}$ ?



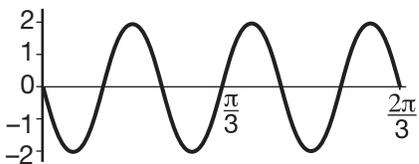
(1)



(3)



(2)



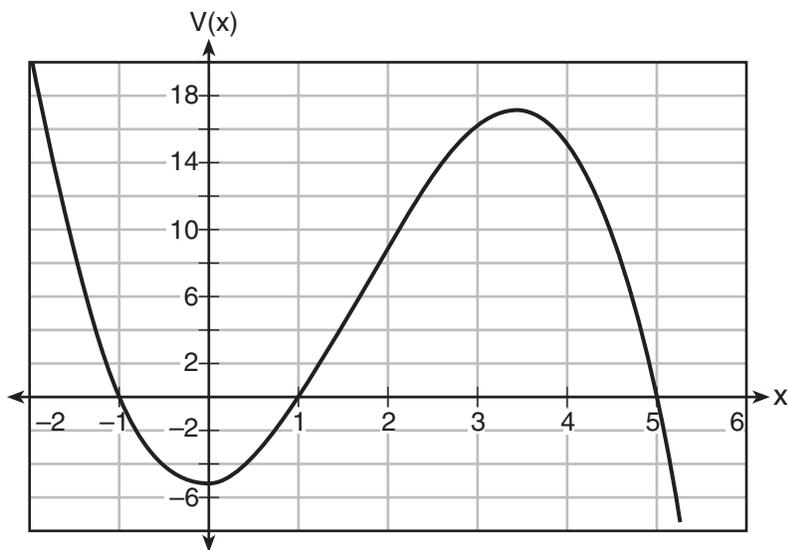
(4)

Use this space for computations.

23 According to a pricing website, Indroid phones lose 58% of their cash value over 1.5 years. Which expression can be used to estimate the value of a \$300 Indroid phone in 1.5 years?

- (1)  $300e^{-0.87}$                       (3)  $300e^{-0.58}$   
(2)  $300e^{-0.63}$                       (4)  $300e^{-0.42}$

24 A cardboard box manufacturing company is building boxes with length represented by  $x + 1$ , width by  $5 - x$ , and height by  $x - 1$ . The volume of the box is modeled by the function below.



Over which interval is the volume of the box changing at the fastest average rate?

- (1)  $[1,2]$                               (3)  $[1,5]$   
(2)  $[1,3.5]$                             (4)  $[0,3.5]$
-

## Part II

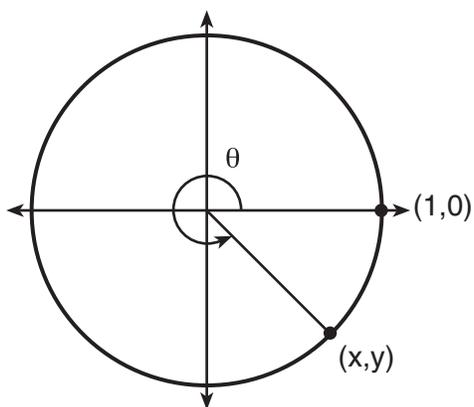
Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Express  $(1 - i)^3$  in  $a + bi$  form.

**26** An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario.

State *one* conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

27 Using the unit circle below, explain why  $\csc\theta = \frac{1}{y}$ .

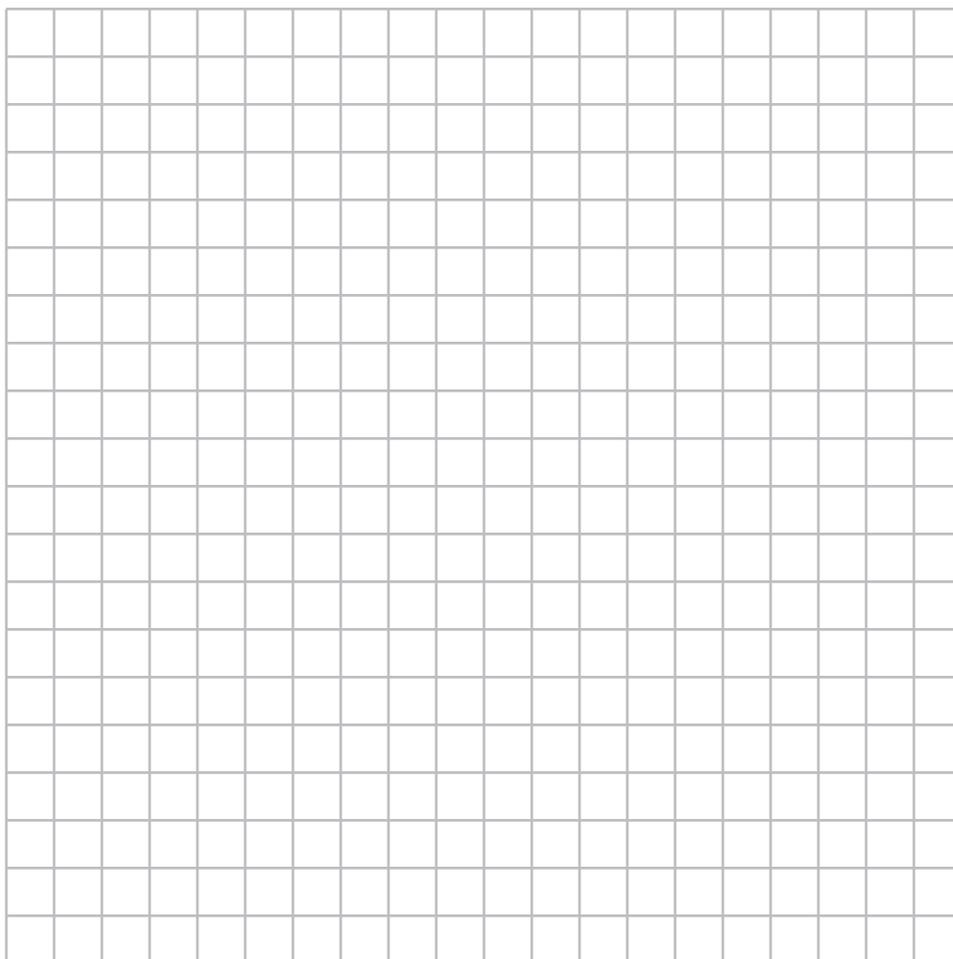


**28** The function  $M(t)$  represents the mass of radium over time,  $t$ , in years.

$$M(t) = 100e^{\frac{\left(\ln \frac{1}{2}\right)t}{1590}}$$

Determine if the function  $M(t)$  represents growth or decay. Explain your reasoning.

**29** On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and  $-2$ .



**30** Given the equal terms  $\sqrt[3]{x^5}$  and  $y^{\frac{5}{6}}$ , determine and state  $y$ , in terms of  $x$ .

**31** The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	<b>Comedy Series</b>	<b>Drama Series</b>	<b>Reality Series</b>	<b>Total</b>
<b>Males</b>	95	65	70	230
<b>Females</b>	80	70	110	260
<b>Total</b>	175	135	180	490

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

**32** Given  $f(x) = 3x^2 + 7x - 20$  and  $g(x) = x - 2$ , state the quotient and remainder of  $\frac{f(x)}{g(x)}$ , in the form  $q(x) + \frac{r(x)}{g(x)}$ .

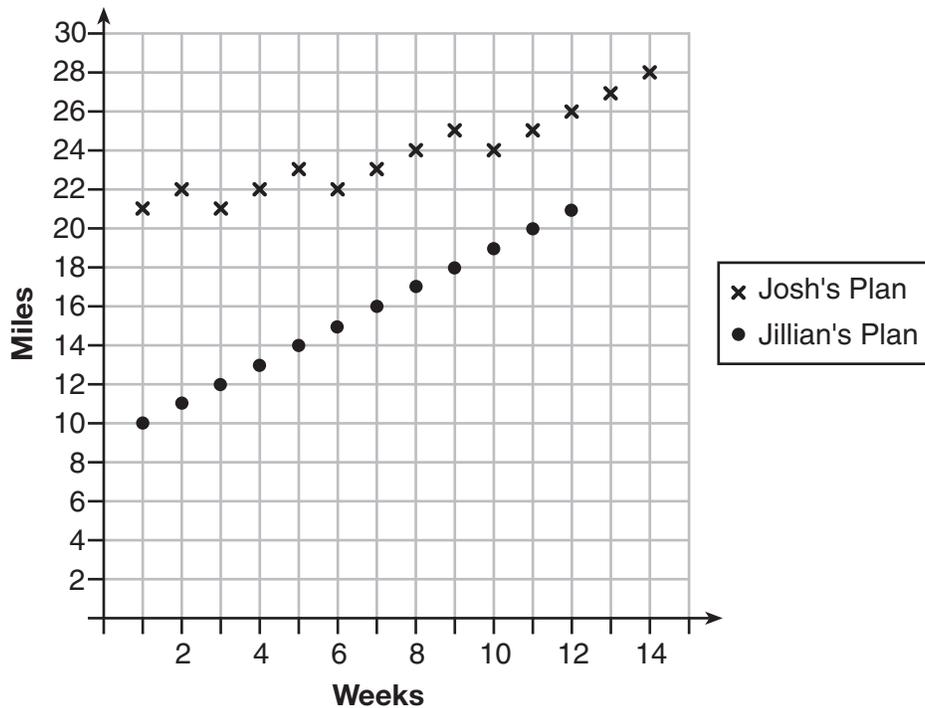
### Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Algebraically determine the values of  $h$  and  $k$  to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian's 12-week plan and Josh's 14-week plan. The number of miles run per week for each plan is plotted below.



Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

Jillian's plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in *simplest form*, to represent the number of miles run each week for the full-marathon training plan.

**35** The guidance department has reported that of the senior class, 2.3% are members of key club,  $K$ , 8.6% are enrolled in AP Physics,  $P$ , and 1.9% are in both.

Determine the probability of  $P$  given  $K$ , to the *nearest tenth of a percent*.

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

**36** Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of \$21,000 and a \$1000 down payment, to the *nearest cent*.

$$P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right)$$

$P_n$  = present amount borrowed

$n$  = number of monthly pay periods

$PMT$  = monthly payment

$i$  = interest rate per month

The affordable monthly payment is \$300 for the same time period. Determine an appropriate down payment, to the *nearest dollar*.

#### Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

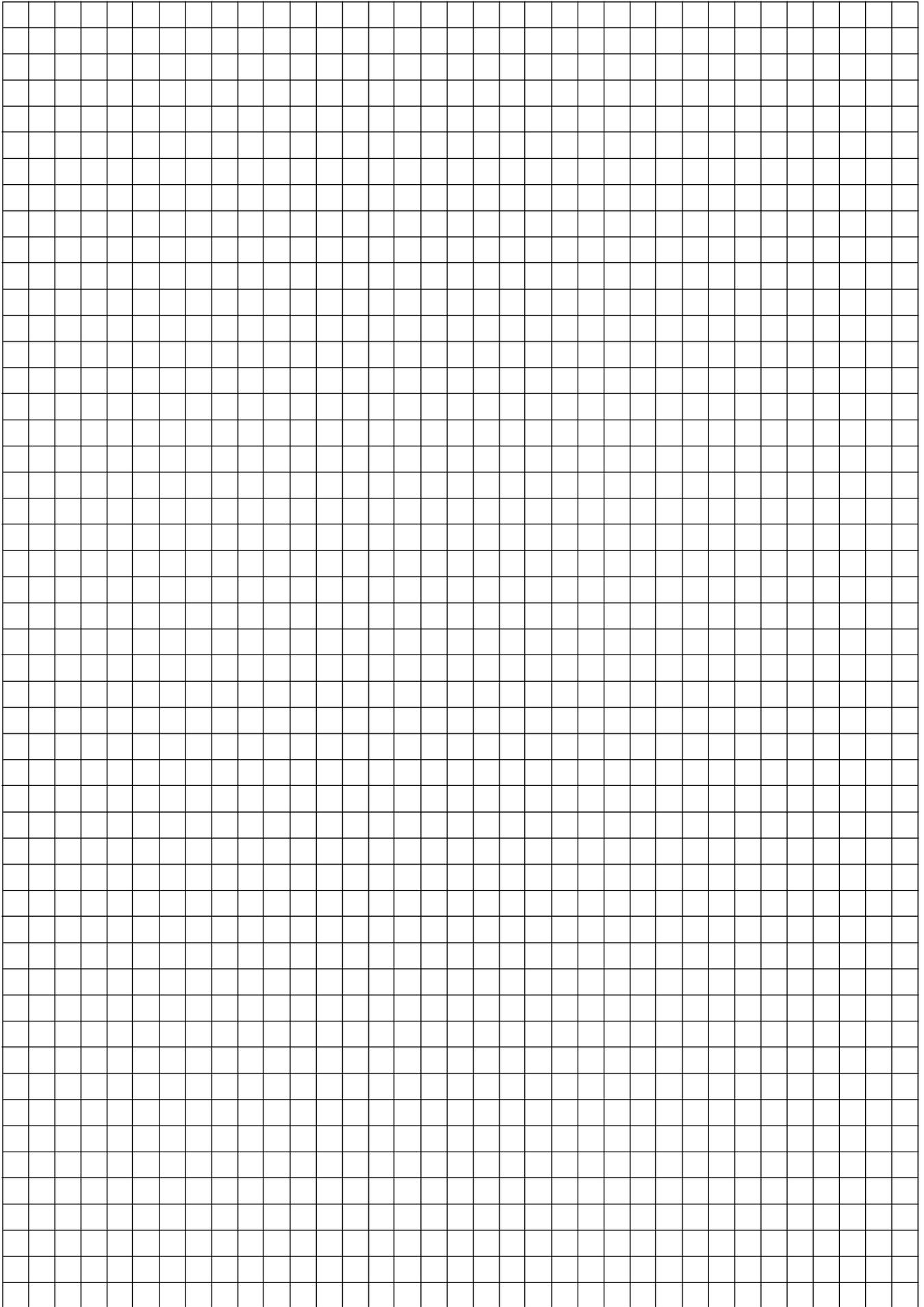
37 The speed of a tidal wave,  $s$ , in hundreds of miles per hour, can be modeled by the equation  $s = \sqrt{t} - 2t + 6$ , where  $t$  represents the time from its origin in hours. Algebraically determine the time when  $s = 0$ .

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the *nearest mile per hour*? Justify your answer.

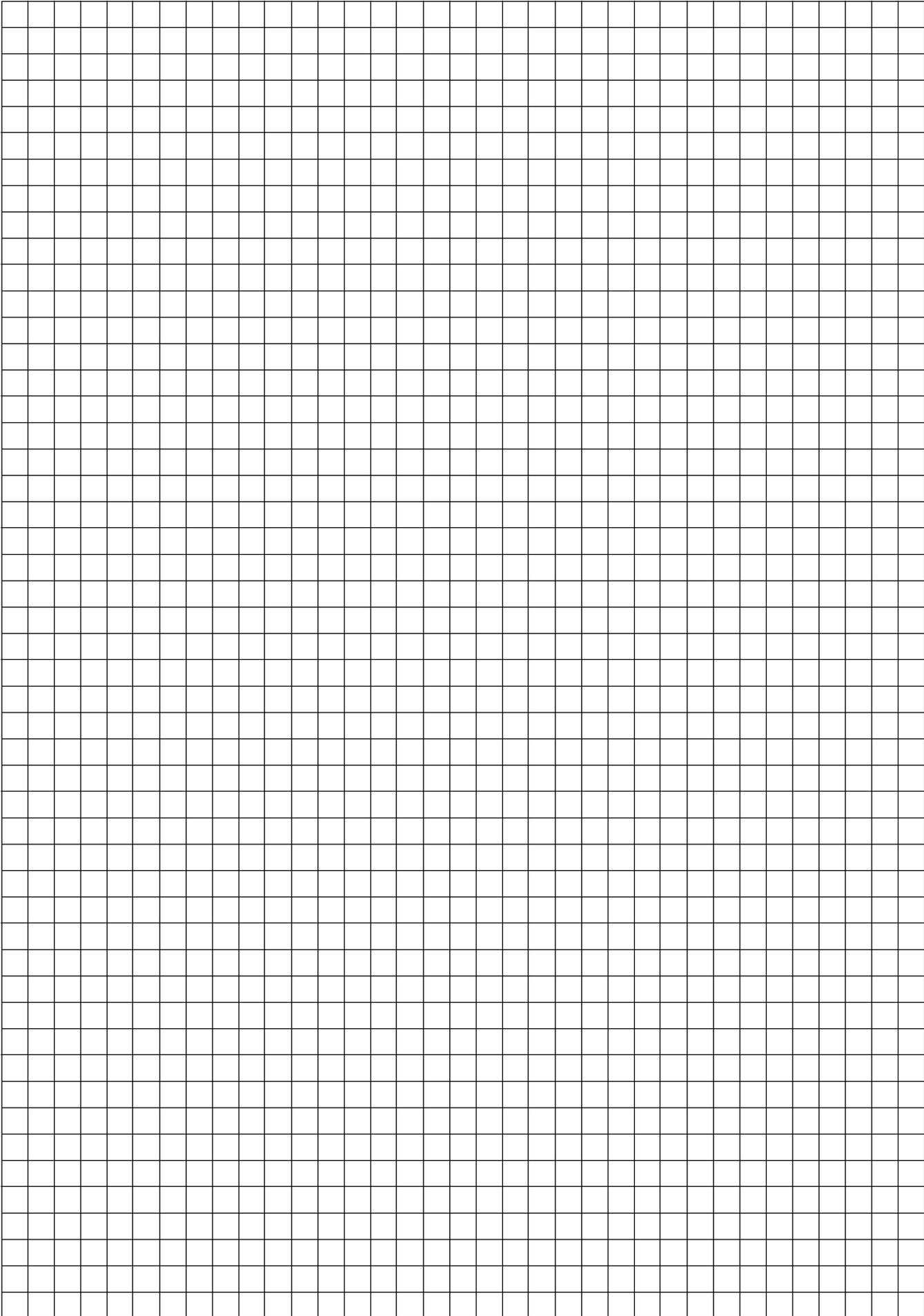
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## High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n - 1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$





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